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AN ANALYSIS OF A GEOMETRY LEARNING PROCESS: THE CASE OF PROVING AREA FORMULAS

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Abstract

Geometry is one of the courses in the curriculum for students of prospective mathematics teachers that can develop deductive thinking ability. The question is, how is the learning and teaching process of the geometry course implemented so as to develop this deductive thinking ability? This research, therefore, aims to investigate the learning and teaching process of a geometry course for prospective mathematics teachers. For reaching these aims, this qualitative study was conducted through observations on the learning process and the written test of a geometry course, for the case of area formulas, involving 56 students of mathematics education program. The results revealed that the learning process is implemented by emphasizing the use of the deductive approach, and from the written test we found various proof strategies in proving an area formula. We conclude that the learning and teaching process of the geometry course has influenced the development of student deductive thinking.

Keywords: Deductive thinking ability, Prospective mathematics teachers, Proving in geometry, Van Hiele theory

Abstrak

Geometri merupakan salah satu mata kuliah untuk mahasiswa calon guru matematika yang dapat mengembangkan kemampuan berpikir deduktif. Pertanyaannya adalah bagaimanakah implementasi proses pembelajaran perkuliahan geometri yang dapat mengembangkan kemampuan berpikir deduktif itu? Oleh karena itu, penelitian ini bertujuan untuk menginvestigasi proses pembelajaran geometri untuk mahasiswa calon guru matematika. Untuk mencapai tujuan tersebut, penelitian kualitatif ini dilakukan melalui observasi proses pembelajaran dan tes tertulis, untuk kasus pembuktian rumus luas daerah bidang datar, yang melibatkan 56 mahasiswa program studi pendidikan matematika. Hasil penelitian menunjukkan bahwa proses pembelajaran geometri yang dilakukan menekankan kepada pembelajaran yang menggunakan pendekatan deduktif, dan dari hasil tes kami temukan beberapa strategi pembuktian berbeda dalam membuktikan rumus luas daerah bidang datar. Kami simpulkan bahwa proses pembelajaran geometri telah mempengaruhi kemampuan berpikir deduktif mahasiswa.

Kata Kunci: Calon guru matematika, Kemampuan berpikir deduktif, Pembuktian dalam geometri, Teori Van Hiele

INTRODUCTION

Geometry, as one of the branches in mathematics (Wallace & West, 1998), is one of the courses in the curriculum for students of prospective mathematics teachers. This geometry course is considered to be able to develop student deductive thinking ability (Hershkowitz, 1998; Howse & Howse, 2015; Jupri, 2018). The deductive thinking ability is indispensable for students of prospective mathematics teachers for pursuing either advanced studies or future careers as mathematics teachers (Jupri & Herman, 2017; Szetela & Nicol, 1992). A natural question that we can pose is, how is the learning and teaching process of the geometry course implemented so as to develop student deductive thinking ability?

For answering the question, this research, therefore, aims to investigate the learning and teaching process of a geometry course for prospective mathematics teachers. In this article, we

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present the results of observations of the learning and teaching of the geometry course for students of prospective mathematics teachers. Theoretical frameworks to analyze the observations include types of learning and teaching approaches and the theory of Van Hiele.

Types of Teaching Approaches

According to Prince and Felder (2006), in general, there are two approaches in the learning and teaching process: inductive and deductive approaches. The deductive approach has a sequence of the learning and teaching process as follows: an explanation of definitions, postulates, concepts, and principles; using definitions, postulates, concepts, and principles to prove theorems and propositions; applying the definitions, theorems, concepts, principles for solving problems or proving other theorems; doing classroom exercises; and conducting an individual test for assessing student understanding. In short, the deductive approach in the learning and teaching process uses deductive thinking, i.e., thinking from general to more specific cases (Mayadiana, 2011; Prince & Felder, 2006; Ruseffendi, 1991; Soedjadi, 2000).

The inductive approach in the learning and teaching process has a sequence as follows: posing a specific problem to be investigated and solved by students; constructing concepts, principles, or formulas through solving the problem; applying the concepts, principles, or formulas to solve other similar problems; and drawing general conclusions based on the learning process. In short, the inductive approach in the learning and teaching process uses inductive thinking, i.e., thinking from specific cases to a more general case (Mayadiana, 2011; Prince & Fleder, 2006; Ruseffendi, 1991; Soedjadi, 2000).

Van Hiele Theory in Geometry Education

According to Van Hiele, student geometric thinking can be classified into five levels: visualization, analysis, abstraction, deduction, and rigor (Breyfogle & Lynch, 2010; Burger & Shaughnessy, 1986; Crowley, 1987; Howse & Howse, 2015; Teppo, 1991; Van Hiele, 1999).
In *level 0: Visualization*, geometric thinking can be characterized by the ability to recognize forms of geometric objects or concepts without considering the properties of the objects. In *level 1: Analysis*, geometric thinking can be indicated by the ability to describe basic geometric concepts by means of an informal analysis of parts and properties. In *level 2: Abstraction*, geometric thinking can be signified by the ability to order properties of geometric concepts: one property follows another property. In the *level 3: Deduction*, the geometric thinking can be indicated by the ability to reason deductively within the context of a mathematical system, that is, to think with undefined terms, postulates, definitions, and theorems. Finally, in *level 4:*

Rigor, geometric thinking can be characterized by the ability to compare different geometry systems.

The above five levels of geometric thinking describe a progression of student thinking from a more concrete visual level to a more sophisticated level of description, analysis, abstraction, and proof (Burger & Shaughnessy, 1986; Van Hiele, 1999). Also, as an important remark, the five levels are sequential, invariant, hierarchical, and the progress depends on the instruction, not age (Clements, 1985; Van Hiele, 1999).

RESEARCH METHOD

To investigate the learning and teaching process of the geometry course for prospective mathematics teachers, we conducted a qualitative study in the form of classroom observations. The observations included two phases. The first phase included an observation of the learning and teaching process on proving area formulas, involving 56 students of mathematics education program in one of the state universities in Bandung, which lasted for 120 minutes. The second phase included an observation of an individual written test on proving a kite area formula, after the first observation, which lasted for 30 minutes.

Data that we collected from the observations included the sequence of the learning process, learning materials, student lecture notes, written student work, and field notes. These data were then analyzed using the framework of types of learning approaches to analyze the learning and teaching process, and the theory of Van Hiele to analyze student geometric thinking.

RESULTS AND DISCUSSION

In this section, we describe two phases of observations: the learning and teaching process; and the result of the written test. Each of these phases is then interpreted using the framework of teaching approaches and the theory of Van Hiele.

The Learning and Teaching Process of Proving Area Formulas

The learning and teaching episode in this observation concerns the area formulas for rectangle, triangle, parallelogram, trapezoid, and kite. From the classroom observation, which lasted for 120 minutes, the learning and teaching process was started by the lecturer by reminding students that they have already known area formulas for rectangle, triangle, and other specific areas in the plane. Even, if they had already known the formulas, they probably did not know yet how to prove the formulas deductively. Therefore, the lecturer then informed students that they would like to learn the idea of area and area formulas according to an axiomatic method. According to Moise (1990), there are at least three postulates on area as a basis for proving theorems about area formulas (see Figure 1).

Postulates for Area

The Congruence Postulate (P1)
If two triangular regions are congruent, then they have the same area.

The Additivity Postulate (P2)
If two polygonal regions intersect only in edges and vertices (or do not intersect at all), then the area of their union is the sum of their areas.

A Square Area Postulate (P3)
If a square region has edges of length a, then its area is a^2.

Theorems for Area Formulas

Theorem 1. The area of a rectangle is the product of its length and its width.
Theorem 2. The area of a right triangle is half the product of the lengths of its legs.
Theorem 3. The area of a triangle is half the product of any base and the corresponding altitude.
Theorem 4. The area of a parallelogram is the product of any base and the corresponding altitude.
Theorem 5. The area of a trapezoid is half the product of the altitude and the sum of the bases.
Theorem 6. The area of a kite is half the products of its diagonals.

Theorem 7. The area of a rhombus is half the product of its diagonals.

Figure 1. Postulates and theorems for area

After explaining the three postulates for area, the lecturer gave examples on how to prove Theorem 1 to Theorem 3. As an example of how to use the postulates for proving the theorems, the proof of Theorem 1 is rewritten and presented in Figure 2.

Next, the lecturer gave an opportunity to students to prove the Theorem 4 as an exercise. In this occasion, the lecturer encouraged students to prove the theorem using postulates and previous theorems, and to use different proving strategies. From the observation of student work and classroom discussion, three different proving strategies for the parallelogram area formula were found. These three proofs are rewritten briefly and presented in Figure 3. A similar discussion occurred for the case of proving the area formula of trapezoid. Theorem 6 was used as a task for an individual written test, which lasted for 30 minutes.

Given a rectangle of length l and width w.

Next, we construct a square of edge $(l+w)$.

According to P3: An area of square

$$(l+w)^2 = l^2 + 2lw + w^2.$$

According to P2:

Area of a square $= A_I + A_{II} + A_{III} + A_{IV}$.

$\Leftrightarrow l^2 + 2lw + w^2 = A_I + l^2 + w^2 + A_{IV}$.

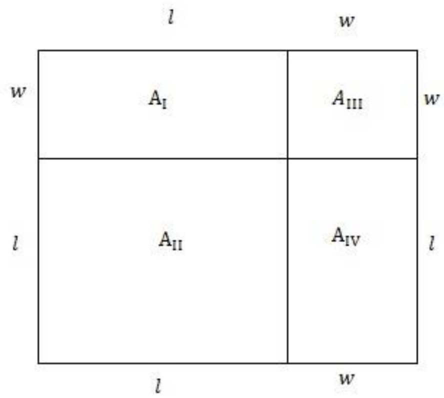
According to P1: $A_I = A_{IV}$.

Therefore, we obtain

$$A_I + A_{IV} = 2lw.$$

$$\Leftrightarrow 2A_I = 2lw.$$

$$\Leftrightarrow A_I = lw.$$



Note: A = Area

Figure 2. The proof of the rectangle area formula

Strategy I

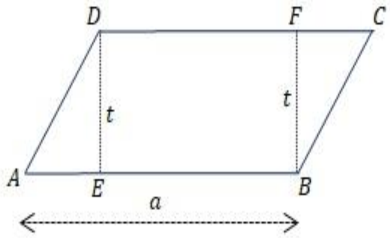
Area of $ABCD = A_{ADE} + A_{BEDF} + A_{BCF}$.

$= \frac{1}{2}AE.t + BE.t + \frac{1}{2}CF.t$

$= \frac{1}{2}t\,(AE + 2BE + CF)$

$= \frac{1}{2}t\,(AE + BE + DF + CF)$

$= at.$



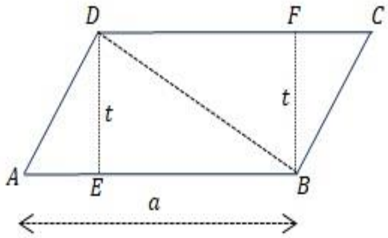
Strategy II

Area of $ABCD = A_{ABD} + A_{BCD}$.

$= \frac{1}{2}AB.t + \frac{1}{2}CD.t$

$= \frac{1}{2}at + \frac{1}{2}at$

$= at.$



Strategy III

Area of $ABCD = A_{AFCE} - A_{AED} - A_{BFC}$.

$= AF.t - \frac{1}{2}DE.t - \frac{1}{2}BF.t$

$= \frac{1}{2}t(AF - DE + AF - BF).$

$= \frac{1}{2}(2a).$

$= at.$

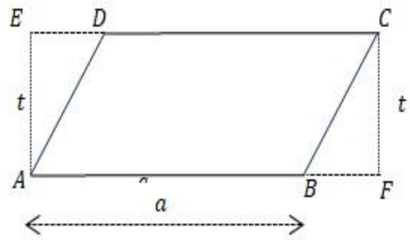


Figure 3. Three different proofs of the parallelogram area formula

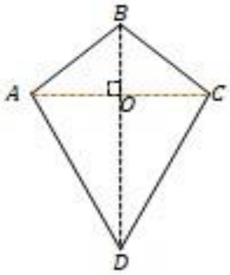
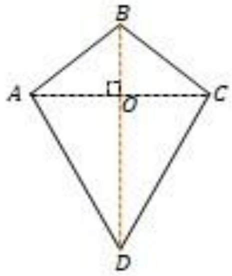
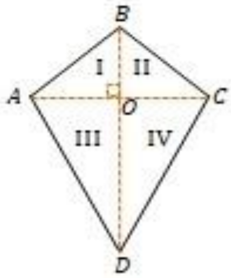
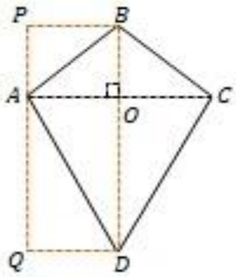
From the above description of the observed learning and teaching process, we can summarize the sequence of the learning process as follows: Explanation about postulates and theorems; giving examples on how to prove area formulas using postulates and theorems; doing classroom exercises and discussion; and conducting a written assessment. In our view, this sequence is considered as a deductive approach in the learning and teaching process (Mayadiana, 2011; Prince & Felder, 2006; Ruseffendi, 1991). The lecturer used this deductive approach for improving student deductive thinking (Soedjadi, 2000). In terms of the Van Hiele theory, this deductive thinking is in line with the deduction level of geometric thinking (Burger & Shaughnessy, 1986; Jupri & Syaodih, 2016; Jupri, 2018; Van Hiele, 1999). A positive note from the learning and teaching process is that even if the lecturer used the deductive approach, which often is characterized as an abstract and rigid way of teaching, still the lecturer motivated students to prove the area formula of parallelogram using various strategies. This process means that the students were encouraged to do an investigative activity that develops critical and creative thinking as well as higher-order thinking skills (As'ari, 2005; Hanna, 2000; Jupri, 2017).

Analysis of Written Test on Proving a Kite Area Formula

The written test required students to prove the kite area formula, was lasted for 30 minutes after the lesson. From the written work of the 56 students of prospective mathematics teachers, we found that 49 students proved the kite area formula correctly. The other seven students did incorrectly or left the worksheet blank (3 students). Also, we found four different strategies for proving this formula. Table 1 presents a brief version of (rewritten) proofs by the students and the corresponding number of students for each type of strategy.

From the perspective of Van Hiele theory, student ability in proving theorems or formulas indicates the level 3 of deduction (Burger & Shaughnessy, 1986; Jupri & Syaodih, 2016; Jupri, 2018; Van Hiele, 1999). This means, in general, that the students of prospective mathematics teachers seem to have reached the deductive level of thinking, which is important for pursuing either advanced studies or careers as mathematics teachers. The finding of various proof strategies can be interpreted as the flexibility of student deductive thinking. This finding seems to be a direct effect of the learning process in which the lecturer provided an opportunity for students to find different strategies in carrying out proving processes.

Table 1. Four different proofs for the kite area formula

Proving Strategy	Number of students (Using the strategy)
Strategy 1 $\text{Area of } ABCD = A_{ABC} + A_{ADC}$ $= \frac{1}{2} AC.BO + \frac{1}{2} AC.DO$ $= \frac{1}{2} AC(BO + OD)$ $= \frac{1}{2} AC.BD.$  	18
Strategy 2 $\text{Area of } ABCD = A_{ABD} + A_{CBD}$ $= \frac{1}{2} BD.AO + \frac{1}{2} BD.CO$ $= \frac{1}{2} BD(AO + OC)$ $= \frac{1}{2} AC.BD.$  	16
Strategy 3 $\text{Area of } ABCD = A_{I} + A_{II} + A_{III} + A_{IV}$ $= \frac{1}{2} AO.BO + \frac{1}{2} CO.BO$ $+ \frac{1}{2} AO.OD + \frac{1}{2} CO.OD.$ $= \frac{1}{2} AO.BD + \frac{1}{2} CO.BD$ $= \frac{1}{2} AC.BD.$  	13
Strategy 4 $\text{Area of } ABCD = \text{Area of } PBDQ$ $= PB.BD$ $= AO.BD$ $= \frac{1}{2} AC.BD.$  	2

CONCLUSION AND RECOMMENDATION

From the results and discussion in the previous section, we draw the following two conclusions. First, the observed learning and teaching process of the geometry course for students of prospective mathematics teachers emphasizes the use of deductive learning and teaching approach. The learning sequence of this approach follows deductive thinking properties, that is, start from more general knowledge, such as the explanation of postulates and theorems, to more specific cases, such as providing examples on how to prove theorems by using postulates and other theorems. The use of this approach is intended for improving student deductive thinking, and in terms of the Van Hiele theory for enhancing student geometric thinking toward the level of deduction. Even if the use of the deductive approach seems abstract and difficult to follow, the lecturer in the observed learning process still provides an opportunity for students to do an investigative activity through the encouragement of finding different strategies for proving theorems. Considering this, for further research, we suggest investigating the impact of the use of an inductive teaching approach toward student deductive thinking ability: whether students will get a better understanding or not through this approach.

Second, the finding of the use of different strategies in proving the kite area formula indicates that the students have used their deductive thinking ability in a flexible manner, depending on their knowledge and experiences, when doing a proving process. For further research, we suggest investigating the effect of investigative activities not only in doing proving processes but also in constructing conjectures through explorative and investigative activities in geometry. In this way, students of prospective mathematics teachers will have balanced experiences on how to learn and to teach geometry using a deductive or inductive approach, which in turn will be useful for their future careers as mathematics teachers and educators.

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