GRAPH TEORY (MA424)

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INTRODUCTION

Definition

A graph G : an ordered triple (V(G), E(G),ψ_G), where

- V(G) is a non empty set of vertices.
- E(G) is a set of edges, disjoint from V(G)
- ψ_G is an incidence function that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G.

Example : G = (V(G), E(G), ψ_G)

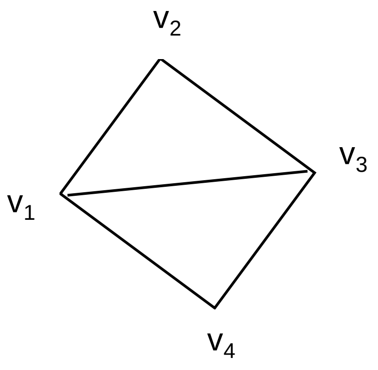
V(G) = $\{v_1, v_2, v_3, v_4\}$

E(G) = $\{e_1, e_2, e_3, e_4, e_5\}$

ψ_G is defined by

$\psi_G(e_1) = v_1v_2$, $\psi_G(e_2) = v_2v_3$, $\psi_G(e_3) = v_3v_4$

$\psi_G(e_4) = v_4v_1$, $\psi_G(e_5) = v_1v_3$



- Two vertices u and v are called adjacent in G if {u, v} is an edge of G.
- If e={u, v}, the edge e is called incident with the vertices u and v.
- The vertices u and v are called endpoints of the edge {u, v}.
- An edge with identical ends is called a loop.
- The edges e_1 and e_2 are called multiple or parallel edges if $\psi_G(e_1) = \psi_G(e_2)$.
- The degree of a vertex v (deg(v)) is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex.
- A vertex of degree zero is called isolated.

The Handshaking Theorem

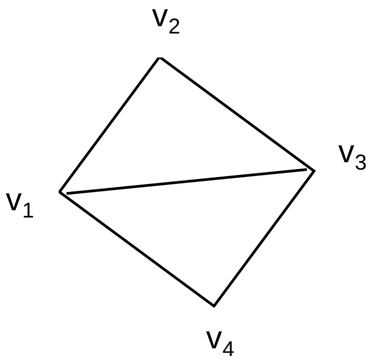
- Let G = (V, E) be an undirected graph with e edges, then

$$\sum_{i=1}^{|V(G)|} \deg_G(v_i) = 2|E(G)|$$

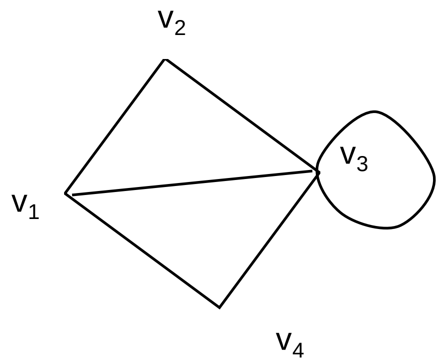
- Corolarry

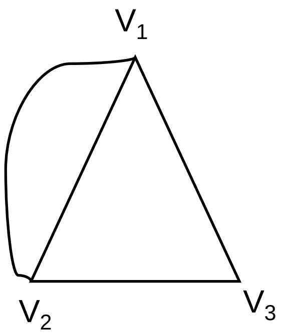
  An undirected graph has an even number of odd degree.

Simple graph

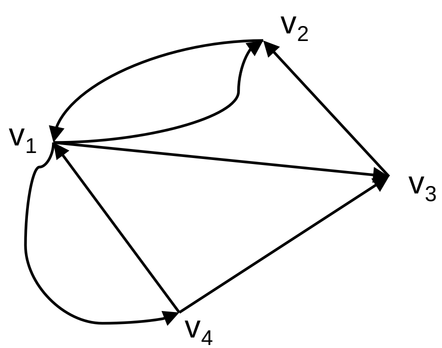


Multigraph





Directed Graph



Graph Application

- Niche overlsp graphs in ecology.
- Round-Robin Tournament.
- The Hollywood Graph.
- Influence Graphs.
- Isomer.

Representing Graph

- Adjacency matrices

  The adjacency matrix of graph *G* is A(*G*) = [a_{ij}], where

  $$a_{ij} = \begin{cases} 1, \text{ if } (v_i,\ v_j) \in E(G) \\ 0, \text{ if } (v_i,\ v_j) \notin E(G) \end{cases}$$

- Incidence matrices

  The incidence matrix of graph *G* is M(*G*) = [m_{ij}] , where

  $$m_{ij} = \begin{cases} 1, \text{ when edge } e_j \text{ is incident with } v_i \\ 0, \quad \text{otherwise} \end{cases}$$

- Diagram
- Adjacency List