# GRAPH TEORY (MATCHINGS)

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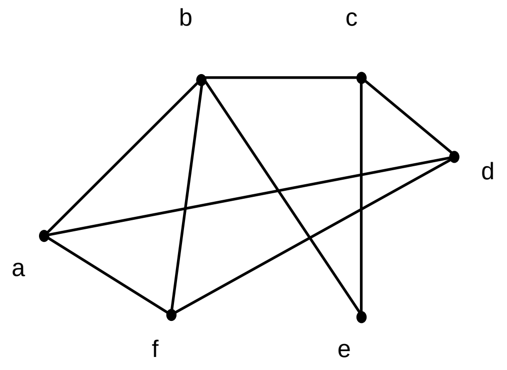
# MATCHINGS

- A subset M of E is called a matching in G if its element are links and no two are adjacent in G.
- The two ends of an edge in M are said to be matched under M.
- A matching M saturates a vertex v, and v is said to be M-saturated, if some edge of M is incident with v, otherwise, v is M-unsaturated.
- Example:

V(G) = {a, b, c, d, e, f}

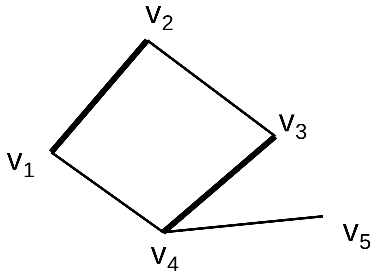
E(G) = {ab, ad, af, bf, be, bc, cd, ce, df}

M = {ab, fd, ce}




*G*

- If every vertex of G is M-saturated, the matching M is perfect.
- M is a maximum matching if G has no matching M’ with |M’|>|M|.
- Every perfect matching is maximum.




Maximum matching

- Let M be a matching in G. An M-alternating path in G is a path whose edges are alternately in E\M and M.
- An M-augmenting path is an M-alternating path whose origin and terminus are M-unsaturated.

- Theorem

  A matching M in G is a maximum matching if and only if G contains no M-augmenting path.

# Matching and covering in Bipartite Graph

- For any set S of vertices in G, the neighbour set of S in G is defined the set of all vertices adjacent to vertices in G; this set is denoted by $N_G(S)$.

- Theorem

  Let G be a bipartite graph with bipartition (X, Y). Then G contains a matching that saturates every vertex in X if and only if

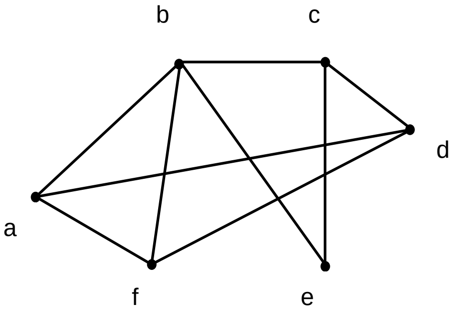
  $$|N(S)| \geq |S| \text{ for all S subset of X.}$$

- Corollary (marriage theorem)

  If G is a k-regular bipartite graph with k>0, then G has a perfect matching.

- A covering of a graph G is a subset K of V such that every edge of G has at least one end in K.
- A covering K is a minimum covering if G has no covering K' with |K'| < |K|.

K = {a, f, b, c} is a covering of G.

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- For any matching M and any covering K, $|M| \leq |K|$.
- Indeed, if M* is a maximum matching and K' is a minimum covering, then $|M^*| \leq |K'|$.

- Lemma

  Let M be a matching and K be a covering such that |M| = |K|. Then M is a maximum matching and K is a minimum covering.

- Theorem

  Let M* is a maximum matching and K’ is a minimum covering. In a bipartite graph |M*| = |K’|.

# Perfect Matching

- A component of a graph is odd or even according as it has an odd or even number of vertices.
- *o*(G) is denoted the number of odd components of G.

- Theorem

  G has a perfect matching if and only if

  $|o(G - S)| \leq |S|$ for all S proper subset of V.