KONTROL H_2 DAN KONTROL H_∞ SERTA APLIKASINYA DALAM SISTEM MASSA PEGAS

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KONTROL H_2 DAN KONTROL H_∞ SERTA APLIKASINYA DALAM SISTEM MASSA PEGAS

- Pendahuluan
- Kontrol H_2
- Kontrol H_∞
- Aplikasi
- Kesimpulan

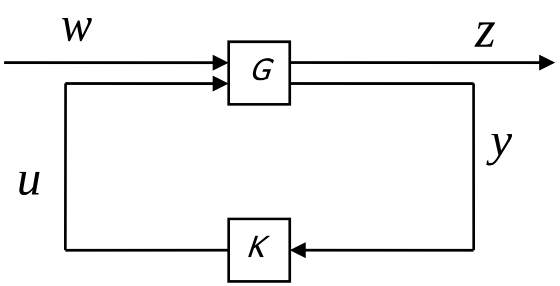
Pendahuluan

Tujuan:

Membandingkan Kontrol H_2 dengan Kontrol H_∞

Kontrol H2

Diagram Sistem Tertutup



Sistem Dinamik

$$\dot{x} = Ax + B_1 w + B_2 u$$
$$z = C_1 x + D_{11} w + D_{12} u$$
$$y = C_2 x + D_{21} w + D_{22} u$$

Kontrol H_2

Matrik transfer:

$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right].$$

Asumsi-asumsi untuk penyederhanaan masalah:

- (A,B_2) terstabilkan dan (C_2,A) terdeteksi;
- $R_1 = D_{12}^* D_{12} > 0 \text{ dan } R_2 = D_{21} D_{21}^* > 0$
- $\begin{bmatrix} A - jwI & B_2 \\ C_1 & D_{12} \end{bmatrix}$ mempunyai rank kolom penuh untuk setiap w
- $\begin{bmatrix} A - jwI & B_1 \\ C_2 & D_{21} \end{bmatrix}$ mempunyai rank baris penuh untuk setiap w

Masalah Kontrol H_2

Masalah utama kontrol H_2 adalah mencari pengontrol K yang proper dan real rational yang menstabilkan G secara internal dan meminimumkan H_2 norm dari transfer matriks Tzw dari w ke z.

Kontrol H_2 (Solusi)

$$H_2 = \begin{bmatrix} A & 0 \\ -C_1^* C_1 & -A^* \end{bmatrix} - \begin{bmatrix} B_2 \\ -C_1^* D_{12} \end{bmatrix} R_1^{-1} \begin{bmatrix} D_{12}^* C_1 & B_2^* \end{bmatrix}$$

$$= \begin{bmatrix} A - B_2 R_1^{-1} D_{12}^* C_1 & -B_2 R_1^{-1} B_2^* \\ -C_1^* (I - D_{12} R_1^{-1} D_{12}^*) C_1 & -(A - B_2 R_1^{-1} D_{12}^* C_1)^* \end{bmatrix}$$

H_2 anggota dom(Ric) dan $X_2 = \mathrm{Ric}(H_2) \geq 0$

$$J_2 = \begin{bmatrix} A^* & 0 \\ -B_1 B_1^* & -A \end{bmatrix} - \begin{bmatrix} C_2^* \\ -B_1 D_{21}^* \end{bmatrix} R_2^{-1} \begin{bmatrix} D_{21} B_1^* & C_2 \end{bmatrix}$$

$$= \begin{bmatrix} (A - B_1 D_{21}^* R_2^{-1} C_2)^* & -C_2^* R_2^{-1} C_2 \\ -B_1 (I - D_{21}^* R_2^{-1} D_{21}) B_1^* & -(A - B_1 D_{21}^* R_2^{-1} C_2) \end{bmatrix}$$

H_2 anggota dom(Ric) dan $Y_2 = \mathrm{Ric}(J_2) \geq 0$

(solusi)

$$F_2 = -R_1^{-1}(B_2^* X_2 + D_{12}^* C_1), \qquad L_2 = -(Y_2 C_2^* + B_1 D_{21}^*) R_2^{-1}$$

$$A_{F2} = A + B_2 F_2, \quad C_{1F2} = C_1 + D_{12} F_2$$

$$A_{L2} = A + L_2 C_2, \quad B_{1L2} = B_1 + L_2 D_{21}$$

$$\hat{A}_2 = A + B_2 F_2 + L_2 C_2$$

$$G_c(s) = \left[\begin{array}{c|c} A_{F2} & I \\ \hline C_{1F2} & 0 \end{array}\right], \quad G_f(s) = \left[\begin{array}{c|c} A_{L2} & B_{1L2} \\ \hline I & 0 \end{array}\right]$$

Terdapat pengontrol optimal $K_{opt}(s) = \left[\begin{array}{c|c} \hat{A}_2 & -L_2 \\ \hline F_2 & 0 \end{array}\right]$

$$\min \|T_{zw}\|_2^2 = \|G_c B_1\|_2^2 + \left\|R_1^{1/2} F_2 G_f\right\|_2^2 = trace(B_1^* X_2 B_1) + trace(R_1 F_2 Y_2 F_2^*)$$

Kontrol H_∞ (Masalah Sederhana)

Matrik transfer:

$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right].$$

Asumsi-asumsi untuk penyederhanaan masalah:

- (A,B_1) terkontrol dan (C_1,A) terobservasi;
- (A,B_2) terstabilkan dan (C_2,A) terdeteksi;
- $D_{12}^{*} \quad C_1 \quad D_{12} \;=\; 0 \quad I \;;$
- $\begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} D_{21}^{*} = \begin{bmatrix} 0 \\ I \end{bmatrix}.$

Kontrol H_∞

Kontrol H_∞

Kontrol Optimal H_∞

Secara Numerik dan Teori Sangat Rumit

SeringTidak Diperlukan

Kontrol Sub Optimal H_∞

- Lebih mudah diperoleh
- Memiliki sifat lebih baik

Kontrol H_∞ (Definisi)

- **Kontrol Optimal**: mencari semua pengontrol K(s), sedemikian sehingga diperoleh $\|T_{zw}\|_\infty$ minimal.
- **Kontrol Suboptimal**: diberikan $, \gamma > 0$ mencari semua pengontrol K(s) yang dapat diterima, sedemikian sehingga $\|T_{zw}\|_\infty < \gamma$

Kontrol H_∞ (Masalah Sederhana)

Matrik transfer:

$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right].$$

Asumsi-asumsi untuk penyederhanaan masalah:

- (A,B_1) terkontrol dan (C_1,A) terobservasi;
- (A,B_2) terstabilkan dan (C_2,A) terdeteksi;
- $D_{12}^{*} \quad C_1 \quad D_{12} = 0 \quad I$;
- $\begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} D_{21}^{*} = \begin{bmatrix} 0 \\ I \end{bmatrix}.$

Kontrol H_∞ (Solusi Hamiltonian)

$$H_\infty := \begin{bmatrix} A & \gamma^{-2} B_1 B_1^* - B_2 B_2^* \\ -C_1^* C_1 & -A_1^* \end{bmatrix}$$

$$J_\infty := \begin{bmatrix} A^* & \gamma^{-2} C_1^* C_1 - C_2^* C_2 \\ -B_1 B_1^* & -A \end{bmatrix}$$

Kontrol H_∞ (Eksistensi Pengontrol Suboptimal)

Teorema : Terdapat suatu pengontrol yang dapat diterima sedemikian sehingga $\|T_{zw}\|_\infty < \gamma$ jika dan hanya jika tiga kondisi berikut terpenuhi:

- $H_\infty \in dom(Ric)$ dan $X_\infty := Ric(H_\infty) \geq 0.$
- $J_\infty \in dom(Ric)$ dan $Y_\infty := Ric(J_\infty) \geq 0.$
- $\rho(X_\infty, Y_\infty) < \gamma^2.$

Kontrol H_∞ (Eksistensi Pengontrol Suboptimal)

Jika ketiga kondisi dipenuhi, maka

$$K_{sub}(s) := \left[\begin{array}{c|c} \hat{A}_\infty & -Z_\infty L_\infty \\ \hline F_\infty & 0 \end{array}\right]$$

dengan

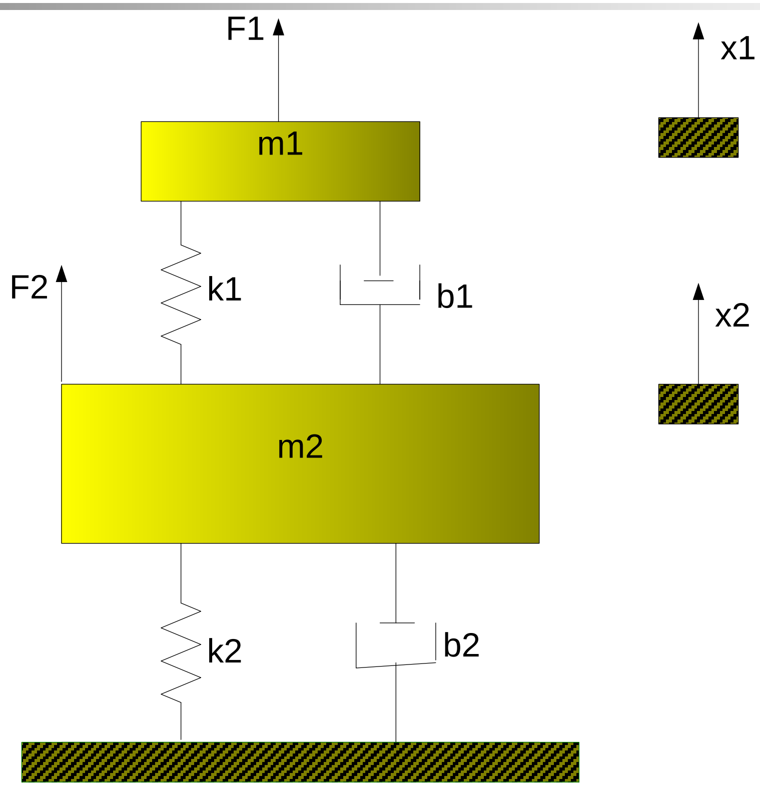
$$\hat{A}_\infty := A + \gamma^{-2} B_1 B_1^* X_\infty + B_2 F_\infty + Z_\infty L_\infty C_2$$

$$F_\infty := -B_2^* X_\infty, \quad L_\infty := Y_\infty C_2^*, \qquad Z_\infty := (I - \gamma^{-2} Y_\infty X_\infty)^{-1}.$$

Perbandingan

Kontrol H_2	Kontrol H_∞
▪ Kontrol optimal H_2 Tunggal ▪ T.12.4 menjamin matriks Hamiltonian H_2 anggota dom(Ric)	▪ Kontrol optimal H_∞ tidak Tunggal untuk sistem MIMO ▪ Blok (1,2) dari matriks Hamiltonian tidak sign definite (Tidak bisa menggunakan T.12.4) ▪ ☎$\to\infty$ maka matriks Hamiltonian H_∞ berkoresponden dengan matriks Hamiltonian H_2

Aplikasi (Sistem Massa Pegas)



Model Sistem Massa Pegas

State-Space Sistem Massa Pegas

- Berdasarkan Hukum Kedua Newton dan hukum kedua Hooke

$$m_1\ddot{x}_1 = b_1(\dot{x}_2 - \dot{x}_1) - k_1x_1 + k_2x_2 + F_1$$

$$m_2\ddot{x}_2 = b_1\dot{x}_1 - (b_1 + b_2)\dot{x}_2 + k_1x_1 + (k_1 + k_2)x_2 + F_2$$

- Jika $x_3 = \dot{x}_1$ dan $x_4 = \dot{x}_2$ maka

$$\dot{x}_3 = -\frac{k_1}{m_1}x_1 + \frac{k_2}{m_1}x_2 - \frac{b_1}{m_1}x_3 + \frac{b_1}{m_1}x_4 + \frac{F_1}{m_1}$$

$$\dot{x}_4 = \frac{k_1}{m_2}x_1 + \frac{k_1 + k_2}{m_2}x_2 + \frac{b_1}{m_2}x_3 - \frac{b_1 + b_2}{m_2}x_4 + \frac{F_2}{m_2}$$

State-Space Sistem Massa Pegas

$$\dot{x}_p = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_1}{m_1} & \dfrac{k_1}{m_1} & -\dfrac{b_1}{m_1} & \dfrac{b_1}{m_1} \\ \dfrac{k_1}{m_2} & -\dfrac{k_1+k_2}{m_2} & \dfrac{b_1}{m_2} & -\dfrac{b_1+b_2}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{1}{m_1} & 0 \\ 0 & \dfrac{1}{m_2} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

State-Space Sistem Massa Pegas

Diberikan nilai:

$$k_1 = 1,\ \ k_2 = 4,\ \ b_1 = 0.2,\ \ b_2 = 0.1,\ m_1 = 1, \text{dan } m_2 = 2$$

State space dapat dinotasikan:

$$\dot{x}_p = A_p x_p + B_p U$$
$$y_p = C_p x_p + D_p U$$

dengan

$$A_p = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -0.2 & 0.1 \\ 0.5 & -2.5 & 0.1 & -0.15 \end{bmatrix},$$

$$C_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$D_p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

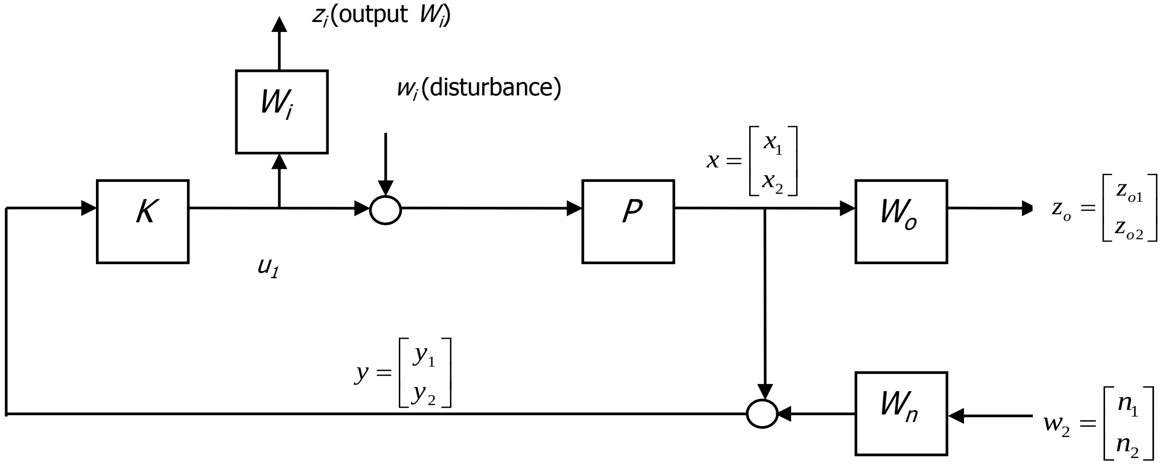
Fungsi Bobot

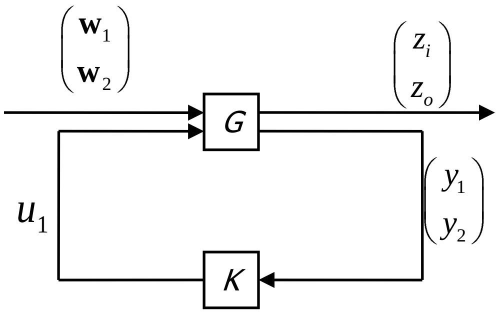
$$W_i = \left[\begin{array}{c|c} A_i & B_i \\ \hline C_i & D_i \end{array}\right] = \left[\begin{array}{cc|cc} -10 & 0 & 1 & 0 \\ 0 & -10 & 0 & 1 \\ \hline 3 & 0 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{array}\right]$$

$$W_o = \begin{bmatrix} \frac{-5}{s+1} & 0 \\ 0 & \frac{-5}{s+1} \end{bmatrix} \Leftrightarrow W_o = \left[\begin{array}{c|c} A_o & B_o \\ \hline C_o & D_o \end{array}\right] = \left[\begin{array}{cc|cc} -1 & 0 & 2.236 & 0 \\ 0 & -1 & 0 & 2.236 \\ \hline -2.236 & 0 & 0 & 0 \\ 0 & -2.236 & 0 & 0 \end{array}\right]$$

$$W_n = \begin{bmatrix} \frac{0.01(s+10)}{s+100} & 0 \\ 0 & \frac{0.01(s+10)}{s+100} \end{bmatrix} \Leftrightarrow W_n = \left[\begin{array}{c|c} A_n & B_n \\ \hline C_n & D_n \end{array}\right] = \left[\begin{array}{cc|cc} -100 & 0 & 0.949 & 0 \\ 0 & -100 & 0 & 0.949 \\ \hline -0.949 & 0 & 0.01 & 0 \\ 0 & -0.949 & 0 & 0.01 \end{array}\right]$$

Blok Diagram dan Sistem Loop Tertutup pada Sistem Massa Pegas





State Space dari Fungsi Bobot

- Untuk W_i

$$\dot{x}_i = A_i x_i + B_i u_1$$
$$z_i = C_i x_i + D_i u_1$$

- Untuk W_o

$$\dot{x}_o = A_o x_o + B_o \mathbf{x} = A_o x_o + B_o (C_p x_p + D_p (u_1 + \mathbf{w}_1))$$
$$z_o = C_o x_o + D_o \mathbf{x} = C_o x_o + D_o (C_p x_p + D_p (u_1 + \mathbf{w}_1))$$

- Untuk W_n

$$\dot{x}_n = A_n x_n + B_n \mathbf{w}_2$$
$$n = C_n x_n + D_n \mathbf{w}_2$$

Untuk P

$$\dot{x}_p = A_p x_p + B_p u_1 + B_p w_1$$
$$y_p = C_p x_p + D_p u_1 + D_p w_1$$

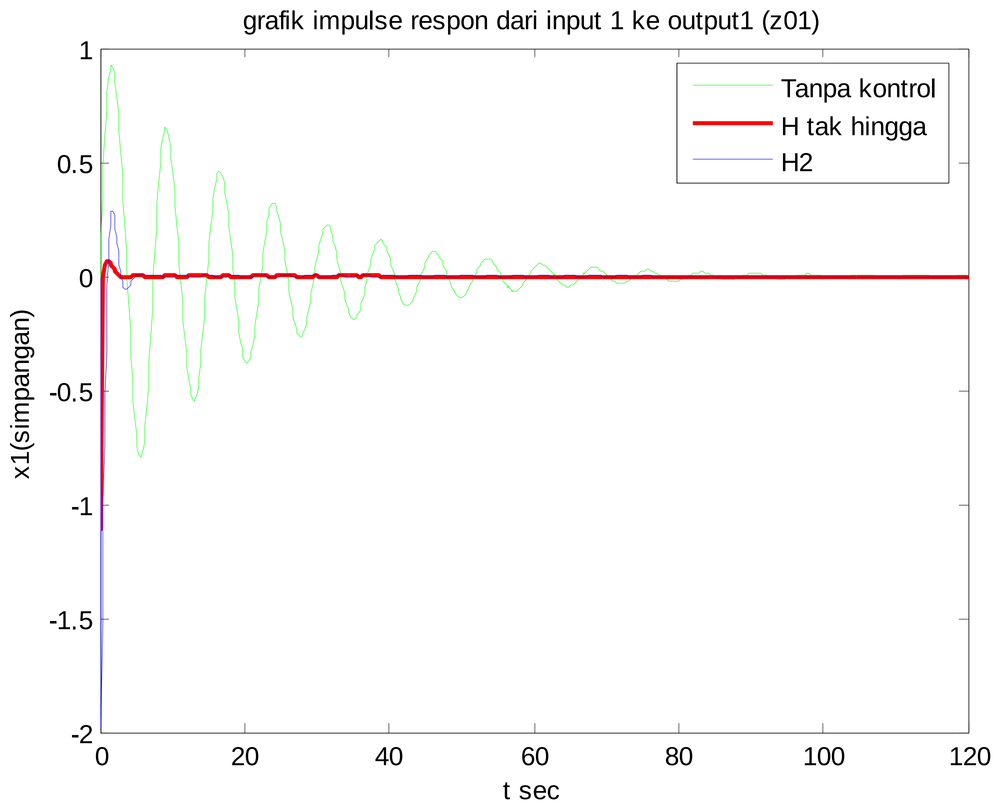
Generalized plant

$$\dot{x} = \begin{bmatrix} \dot{x}_i \\ \dot{x}_p \\ \dot{x}_o \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} A_i & 0 & 0 & 0 \\ 0 & A_p & 0 & 0 \\ 0 & B_o C_p & A_o & 0 \\ 0 & 0 & 0 & A_n \end{bmatrix} \begin{bmatrix} x_i \\ x_p \\ x_o \\ x_n \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_p & 0 \\ D_p & 0 \\ 0 & B_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} B_i \\ B_p \\ D_P \\ 0 \end{bmatrix} u_1$$

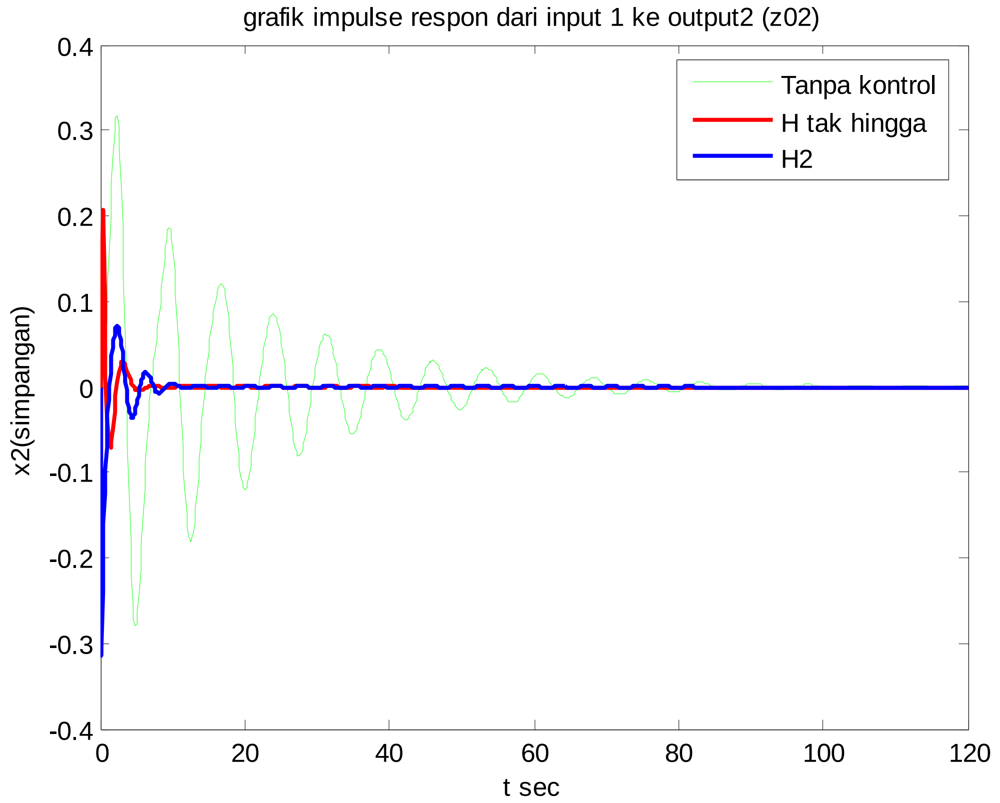
$$z = \begin{bmatrix} z_i \\ z_o \end{bmatrix} = \begin{bmatrix} C_i & 0 & 0 & 0 \\ 0 & D_o C_p & C_o & 0 \end{bmatrix} \begin{bmatrix} x_i \\ x_p \\ x_o \\ x_n \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ D_o D_p & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} D_i \\ D_o D_p \end{bmatrix} u_1 \quad \Longleftrightarrow \quad G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

$$y = x + n = \begin{bmatrix} 0 & C_p & 0 & C_n \end{bmatrix} \begin{bmatrix} x_i \\ x_p \\ x_o \\ x_n \end{bmatrix} + \begin{bmatrix} D_p & D_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + D_p u_1$$

Hasil



grafik impulse respon dari input 1 ke output1 (z01)
Tanpa kontrol
H tak hingga
H2
x1(simpangan)
t sec
1
0.5
0
-0.5
-1
-1.5
-2
0
20
40
60
80
100
120



grafik impulse respon dari input 1 ke output2 (z02)
Tanpa kontrol
H tak hingga
H2
x2(simpangan)
t sec
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
20
40
60
80
100
120

Hasil

- Untuk kontrol H_2 dengan nilai norm:

$$\|T_{zw}\|_2 = 1.7361$$

- Untuk kontrol H_∞ dengan nilai norm:

$$\|T_{zw}\|_\infty = 0.9515$$