Fuzzy System

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Latar Belakang

- Pada hidup sehari-hari, kita terbiasa dengan ucapan “kecil”, “agak panas”, “sekitar jam 2”.
- Ucapan yang tidak presisi (imprecise term) susah dimodelkan dalam classical mathematics.

Aplikasi

- Tahun 1990, pertama kali dibuat mesin cuci dengan fuzzy logic di jepang (Matsushita Electric Industrial Company). Sistem digunakan untuk menentukan putaran yang tepat secara otomatis berdasarkan jenis dan banyaknya kotoran serta jumlah yang akan dicuci. Input adalah seberapa kotor, jenis kotoran, dan banyaknya yang dicuci. Mesin ini menggunakan sensor optik, mengeluarkan cahaya ke air dan mengukur bagaimana cahaya tersebut sampai keujung lainnya.
- Transmisi otomatis pada mobil. Mobil Nissan menggunakan sistem ini dan dapat menghemat bensin 12-17%.
- Kereta bawah tanah sendai mengontrol pemberhentian otomatis pada area tertentu.
- Ilmu kedokteran dan biologi, seperti sistem diagnosis, penelitian kanker.
- Klasifikasi dan pencocokan pola.

Fuzzy Logic dan Fuzzy Set

- Fuzzy logic is a generalized kind of logic
- Fuzzy sets are the key to the semantics of vague linguistic terms
- Fuzzy sets are based on fuzzy logic

To have a solid basis for studying fuzzy logic and fuzzy sets, a short overview of **classical logic and set** theory is necessary.

Classical Logic

Classical Two-Valued Logic

- The set of truth values consists of two elements: $\{0, 1\}$
- There are two basic binary operations $\wedge, \vee$ and the unary complement $\neg$
- Other logical operations, e.g. the implication $\rightarrow$, the logical equivalence $\leftrightarrow$, and the exclusive or, can be constructed from the three basic operations $\wedge, \vee, \neg$

Classical Logic (cont'd)

Propositional Logic

- Propositions are either true or false
- A proposition may either be an atomic propositional variable $p_1, p_2, \ldots$ or a compound expression $(p \wedge q)$, $(p \vee q)$, or $\neg p$, where p and q are propositions
- The truth value of a proposition is evaluated by assigning a truth value to each propositional variable and evaluating the formula "from the inside to the outside" applying the logical operations

Classical Logic (cont'd)

Truth Tables of the Three Basic Logical Operations

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

p	$\neg p$
0	1
1	0

Classical Logic (cont'd)

Basic Properties

The following holds for all assignments of p, q, and r:

1. $p \wedge q = q \wedge p$, $p \vee q = q \vee p$ (commutativity)
2. $p \wedge (q \wedge r) = (p \wedge q) \wedge r$, $p \vee (q \vee r) = (p \vee q) \vee r$ (associativity)
3. $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$, $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$ (distributivity)
4. $p \wedge 1 = p$, $p \vee 0 = p$ (neutral elements)
5. $p \wedge 0 = 0$, $p \vee 1 = 1$ (absorption)
6. $p \wedge p = p$, $p \vee p = p$ (idempotence)
7. $\neg(\neg p) = p$ (involution)
8. $\neg(p \wedge q) = \neg p \vee \neg q$, $\neg(p \vee q) = \neg p \wedge \neg q$ (De Morgan laws)
9. $p \wedge \neg p = 0$, $p \vee \neg p = 1$ (excluded middle)

Classical Sets

- Pada teori himpunan klasik atau classical sets, suatu himpunan adalah kumpulan elemen-elemen yang berbeda. Himpunan klasik disebut juga **crisp set**.
- Crisp set diartikan sebagai clear and distinct yaitu himpunan yang membedakan anggota dan non-anggota dengan jelas.
  - Misal, $A=\{x|x$ bilangan bulat, $x>6\}$ maka 6,5, … bukan anggota himpunan.

Classical Set (cont'd)

Basics of Set Theory

- A set A is a collection of objects belonging to a given universe X, where, for each possible object x from the universe X, it is decidable whether it belongs to the set A or not; if so, we say $x \in A$; if not, we say $x \notin A$
- A set A is a subset of B if all objects in A are in B as well; if so, we say $A \subseteq B$; we say $A \subset B$ if there is at least one element in B which is not in A
- The set of all subsets of X is denoted with $\mathcal{P}(X)$
- The empty set, which does not contain any object, is denoted with $\emptyset$

Classical Set (cont'd)

Operations on Sets

- The intersection of two sets A and B is the set of objects from X which belong both to A and B; we denote this set with $A \cap B$
- The union of two sets A and B is the set of objects from X which belong at least to one of the sets A and B; we denote this set with $A \cup B$
- The complement of a set A is the set of objects from X which do not belong to A; we denote this set with $\complement A$

Classical Set (cont'd)

Connection to Logic

- $A \cap B = \{x \in X \mid x \in A \wedge x \in B\}$
- $A \cup B = \{x \in X \mid x \in A \vee x \in B\}$
- $\complement A = \{x \in X \mid x \notin A)\} = \{x \in X \mid \neg(x \in A)\}$

- $A \subseteq B$ if and only if $(x \in A) \rightarrow (x \in B)$ for all $x \in X$

Classical Set (cont'd)

Basic Properties

The following holds for all sets A, B, C on the same universe X:

1. $A \cap B = B \cap A$, $A \cup B = B \cup A$ (commutativity)
2. $A \cap (B \cap C) = (A \cap B) \cap C$, $A \cup (B \cup C) = (A \cup B) \cup C$ (associativity)
3. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (distributivity)
4. $A \cap X = A$, $A \cup \emptyset = A$ (neutral elements)
5. $A \cap \emptyset = \emptyset$, $A \cup X = X$ (absorption)
6. $A \cap A = A$, $A \cup A = A$ (idempotence)
7. $\complement(\complement A) = A$ (involution)
8. $\complement(A \cap B) = \complement A \cup \complement B$, $\complement(A \cup B) = \complement A \cap \complement B$ (De Morgan laws)

Classical Set (cont'd)

Characteristic Function

The characteristic function of a set A is defined as follows (for all $x \in X$):

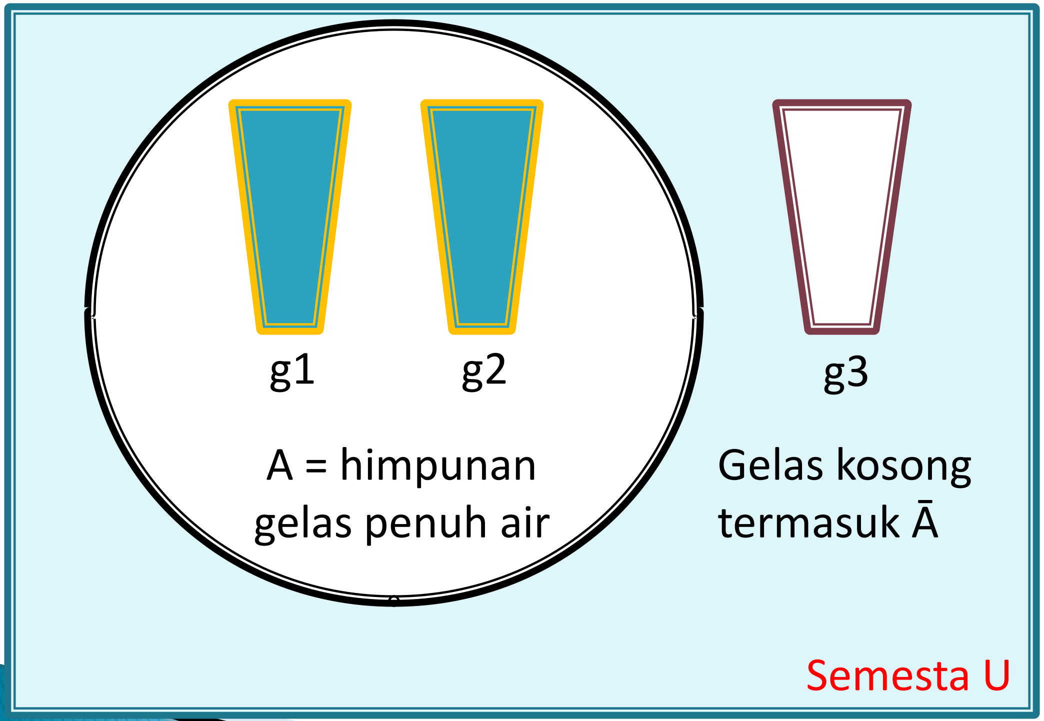
$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

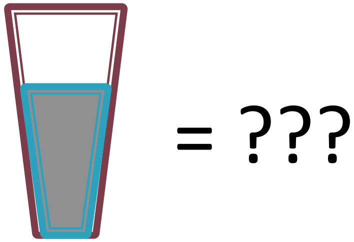
Connection to logic:

- $\chi_{A \cap B}(x) = \chi_A(x) \wedge \chi_B(x)$
- $\chi_{A \cup B}(x) = \chi_A(x) \vee \chi_B(x)$
- $\chi_{\complement A}(x) = \neg \chi_A(x)$
- $A \subseteq B$ if and only if $(\chi_A(x) \rightarrow \chi_B(x)) = 1$ for all $x \in X$

Keterbatasan Classical Set

- Pada Excluded Middle Laws dinyatakan bahwa suatu elemen harus termasuk dalam A atau Ā.





Gelas yang berisi air setengah bagian tidak termasuk A maupun Ā

Fuzzy Set

What are Fuzzy Sets?

The idea behind fuzzy logic is to replace the set of truth values $\{0, 1\}$ by the entire unit interval $[0, 1]$.

A fuzzy set on a universe X is represented by a function which maps each element $x \in X$ to a degree of membership from the unit interval $[0, 1]$. These so-called membership functions are direct generalizations of characteristic functions.

Contoh Kasus:

Suhu (x ϵ X)	Derajat Keanggotaan (degree of membership)		
	Dingin	Hangat	Panas
5	1	0.1	0
15	0.9	0.8	0
25	0.5	1	0.6
35	0.1	0.6	0.9
45	0	0.2	1

Pada tabel, 5^o c dikatakan sungguh-sungguh dingin dengan tingkat kebenaran 1, sedangkan 25^o c termasuk hangat, dst. Dituliskan dalam fuzzy set:

Dingin = {5, 15, 25, 35} dan derajat keanggotaannya dinyatakan oleh μ_{Dingin} = {1;0.9;0.5;0.1}.

Hangat = {5, 15, 25, 35, 45} dan derajat keanggotaannya dinyatakan oleh μ_{Hangat} = {0.1;0.8;1;0.6;0.2}.

Panas = {25, 35, 45} dan derajat keanggotaannya dinyatakan oleh μ_{Panas} = {0.6;0.9;1}

Representasi Fungsi Keanggotaan

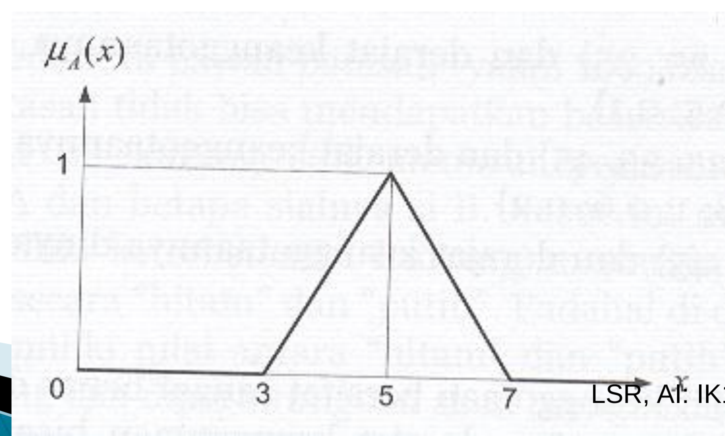
- Tabular and list:

  Dingin = {<5,1>,<15,0.9>,<25,0.5>,<35,0.1>} atau

  Dingin = 1/5+0.9/15+0.5/25+0.1/35
- Geometric: direpresentasikan dalam ruang euclidean n-dimensional: element (suhu, °c, derajat keanggotaan) = (dingin, 5, 1)
- Analytic:

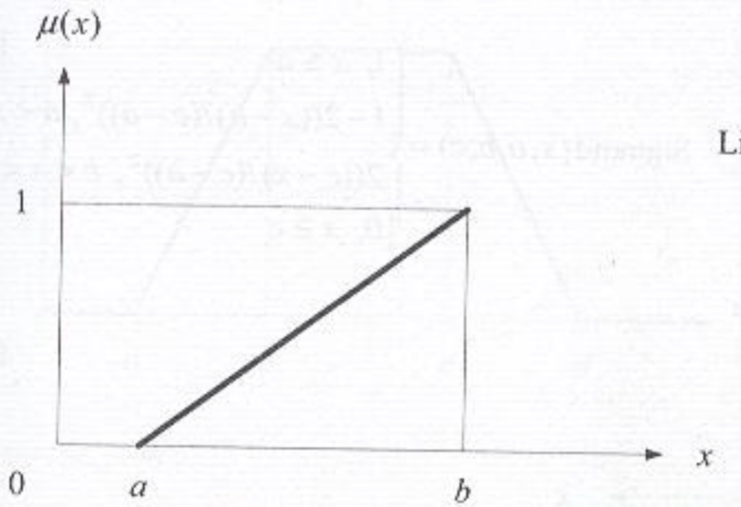
$$\mu_A(x) = \begin{cases} x-3 & untuk \quad 3 \le x \le 5 \\ 7-x & untuk \quad 5 \le x \le 7 \\ 0 & untuk\ lainnya \end{cases}$$

- Graphical:

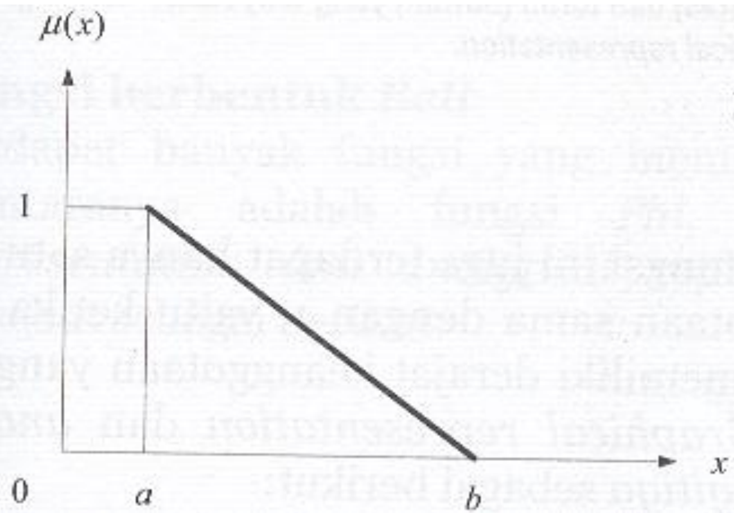


Fungsi Keanggotaan

- Fungsi Linear



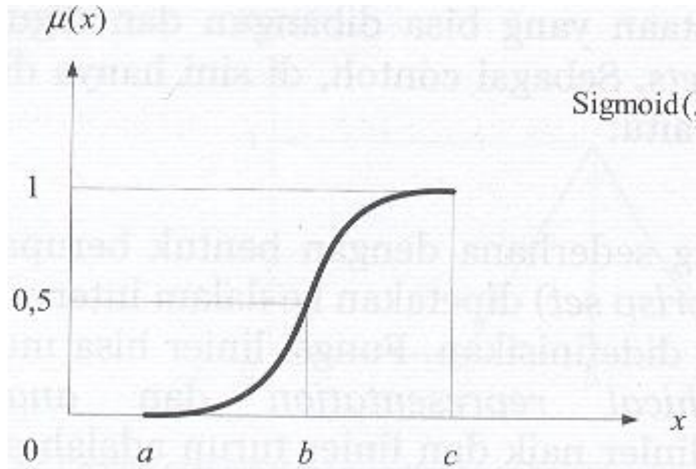
$$\text{LinierNaik}(x,a,b)=\begin{cases}0, & x\le a\\ (x-a)/(b-a), & a<x\le b\end{cases}$$



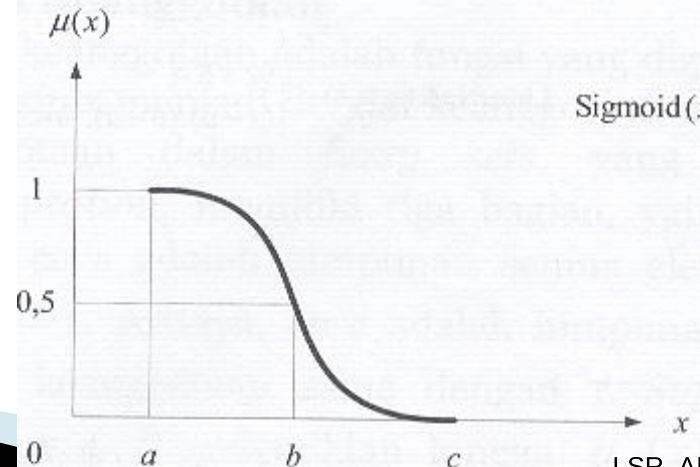
$$\text{LinierTurun}(x,a,b)=\begin{cases}(b-x)/(b-a), & a\le x<b\\ 0, & x\ge b\end{cases}$$

Fungsi Keanggotaan (cont'd)

- Fungsi Sigmoid



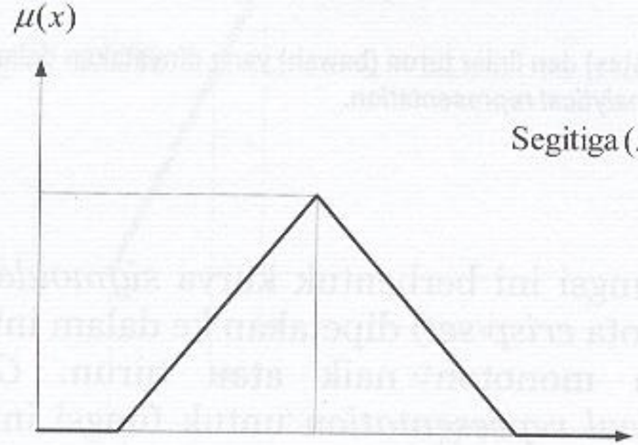
$$\text{Sigmoid}(x,a,b,c)=\begin{cases}0, & x\le a\\ 2((x-a)/(c-a))^2, & a<x\le b\\ 1-2((c-x)/(c-a))^2, & b<x<c\\ 1, & c\le x\end{cases}$$



$$\text{Sigmoid}(x,a,b,c)=\begin{cases}1, & x\le a\\ 1-2((x-a)/(c-a))^2, & a<x\le b\\ 2((c-x)/(c-a))^2, & b<x<c\\ 0, & x\ge c\end{cases}$$

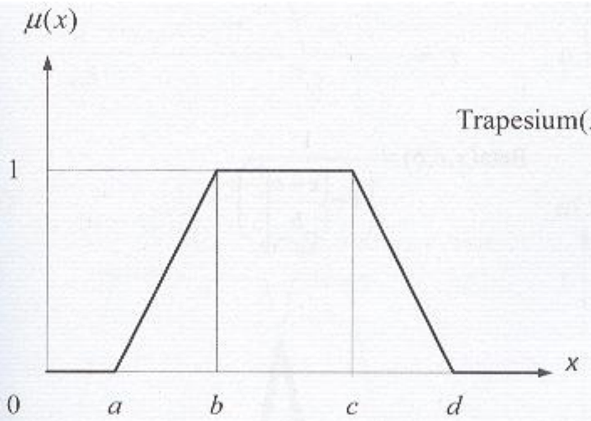
Fungsi Keanggotaan (cont'd)

- Fungsi Segitiga



$$\text{Segitiga}(x,a,b,c)=\begin{cases} 0, x \le a, x \ge c \\ (x-a)/(b-a),\ a < x \le b \\ -(x-c)/(c-b), b < x \le c \end{cases}$$

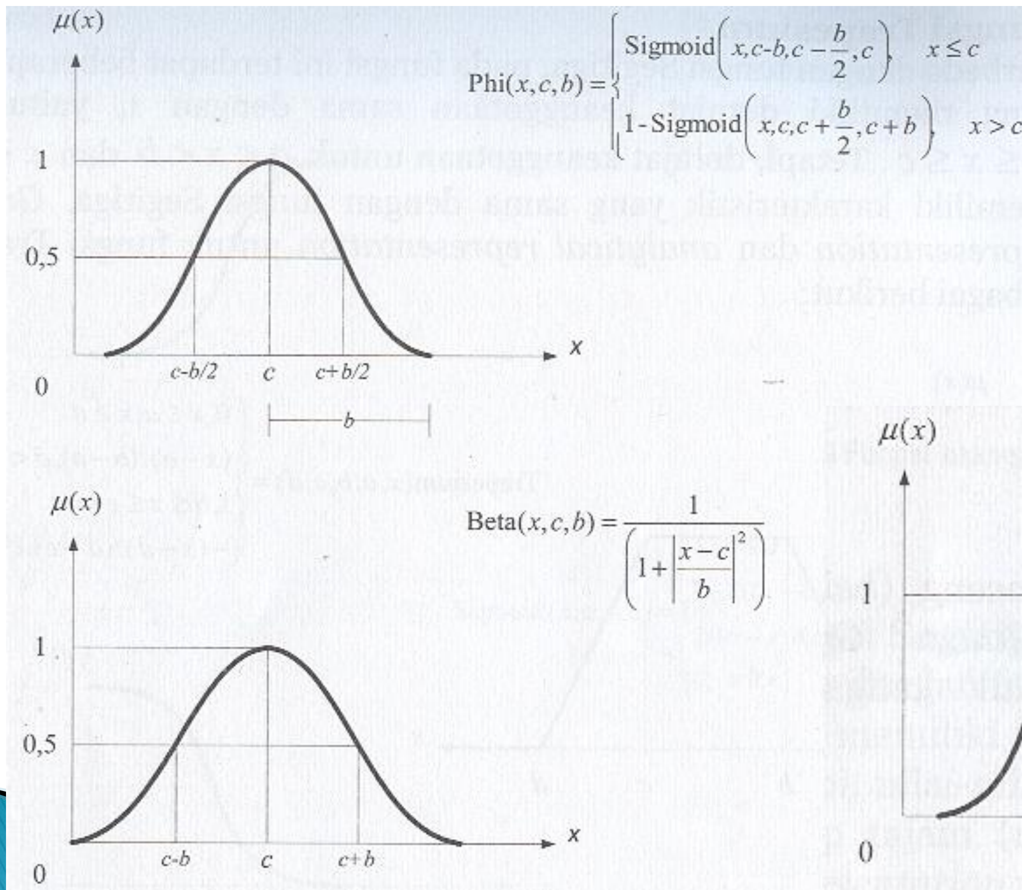
- Fungsi Trapesium

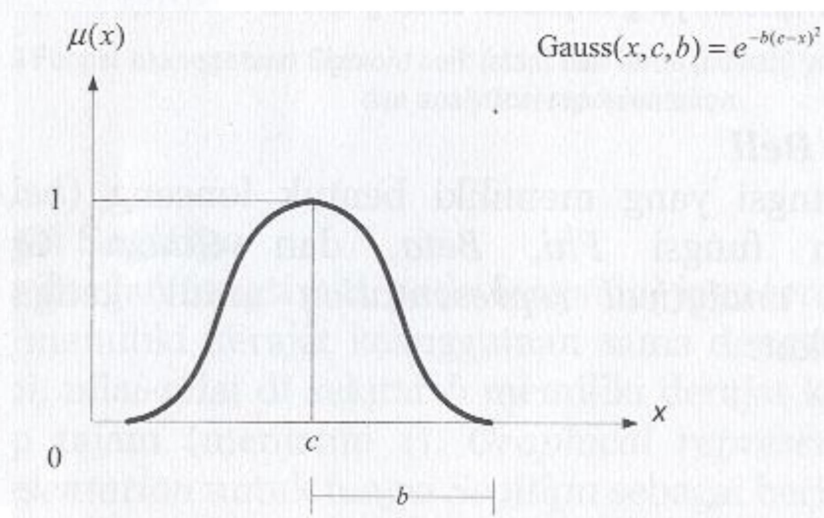


$$\text{Trapesium}(x,a,b,c,d)=\begin{cases} 0, x \le a, x \ge d \\ (x-a)/(b-a), a < x < b \\ 1, b \le x \le c \\ -(x-d)/(d-c), c < x \le d \end{cases}$$

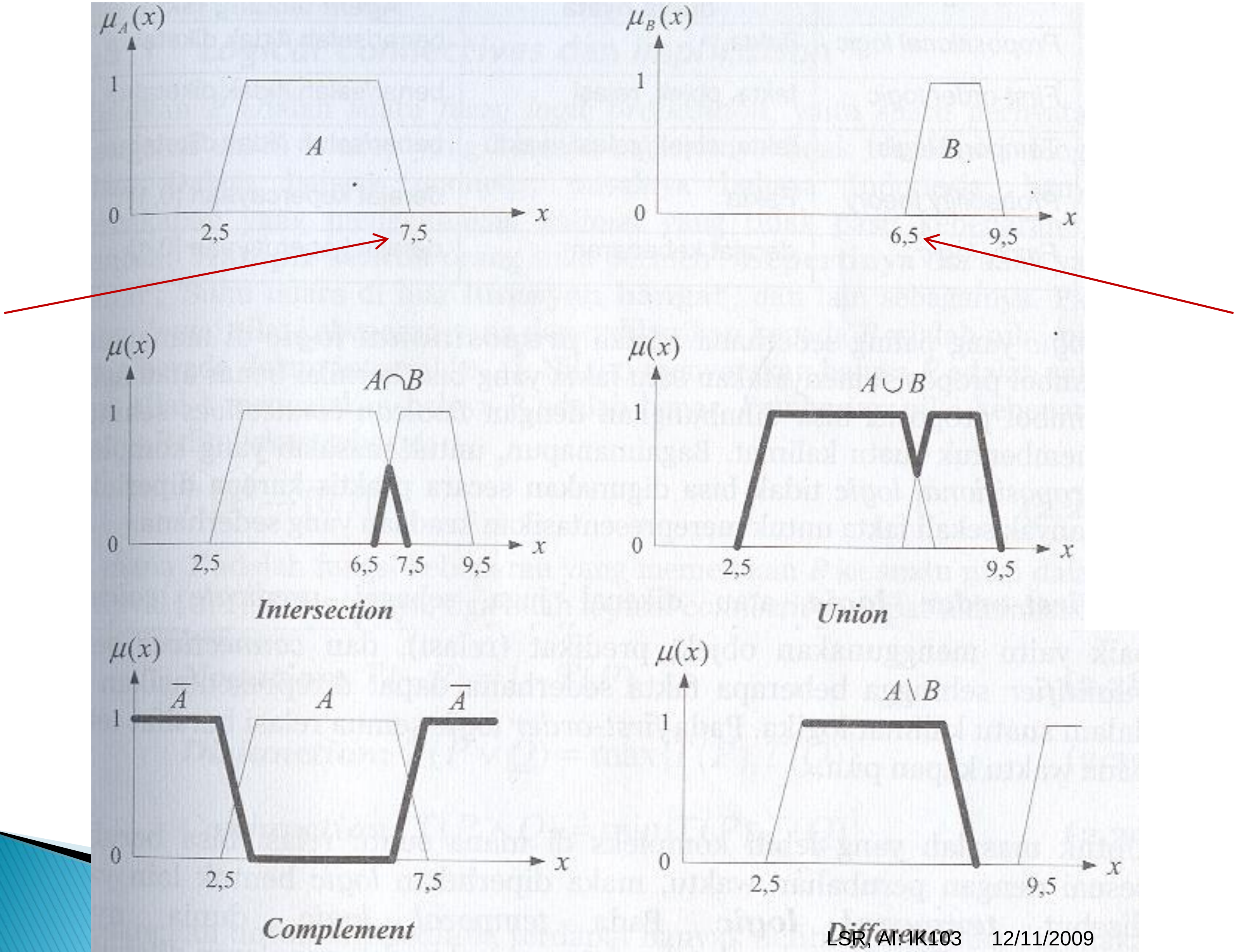
Fungsi Keanggotaan (cont'd)

- Fungsi Bell

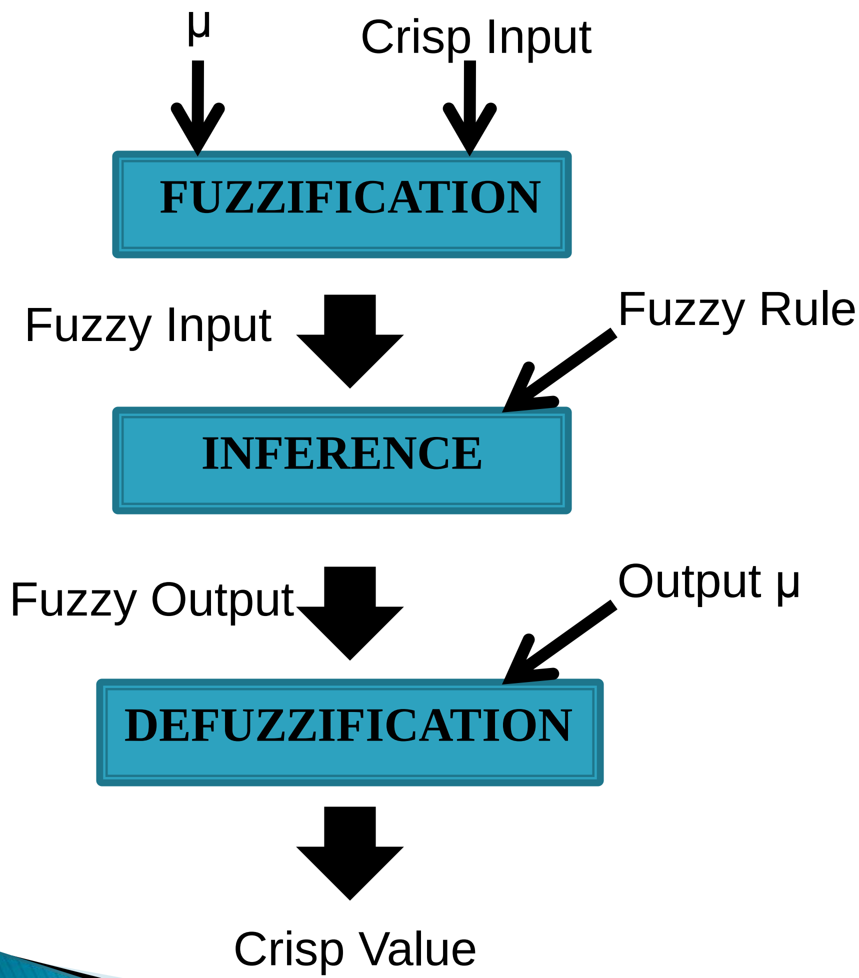




Operasi Fuzzy Sets



Fuzzy Sistem



- Fuzzification: Mengubah masukan dari nilai kebenaran bersifat pasti → fuzzy input.
- Inference: Penalaran menggunakan fuzzy input dan fuzzy rule yang telah didefinisikan.
- Defuzzification: Mengubah fuzzy output → crisp value

Masalah: Pemberian Beasiswa

- Misalkan suatu program pemberian beasiswa kepada mahasiswa di suatu perguruan tinggi X dilakukan berdasarkan dua kriteria: IPK dan Gaji perbulan orang tuannya (G). Beasiswa diberikan kepada mahasiswa yang memiliki prestasi akademik bagus tetapi tingkat ekonomi orang tuanya rendah. Misalkan terdapat dua mahasiswa, si A dan si B, dengan data- data seperti pada tabel.

Mahasiswa	IPK	Gaji (Rp./bulan)
A	3,00	10 juta
B	2.99	1 juta

Dari dua mahasiswa tersebut siapa yang mendapatkan beasiswa ???

Langkah Fuzzy Logic

1. FUZZIFICATION: Menentukan representasi fungsi keanggotaan pada tiap element (IPK dan gaji).
2. INFERENCE: Menentukan model fuzzy rule
   a. Model Mamdani IF x_1 is A_1 AND …AND x_n is A_n THEN y is B.
   b. Model Sugeno IF x_1 is A_1 AND… AND x_n is A_n THEN $y=f(x_1,\ldots x_n)$
      dengan $f(x_1,\ldots,x_n=)=w_0+w_1x_1+\ldots+w_nx_n$

3. DEFUZZIFICATION: (Centroid Method, Height Method, First or Last of Maxima, Mean Max Method, Weighted Average)

1. FUZZIFICATION

- Fungsi keanggotaan IPK:

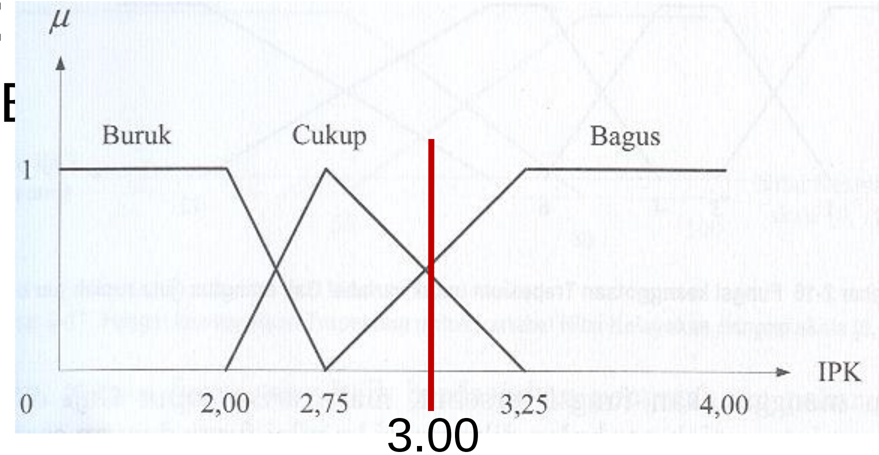
IPK 3.00: Nilai liguistik “Cukup” & “B

Cukup: μ(x)=−(x−c)/(c−b) =

−(3−3.25)/(3.25−2.75)=0.5

Bagus: μ(x)=(x−a)/(b−a) =

(3−2.75)/(3.25−2.75) = 0.5



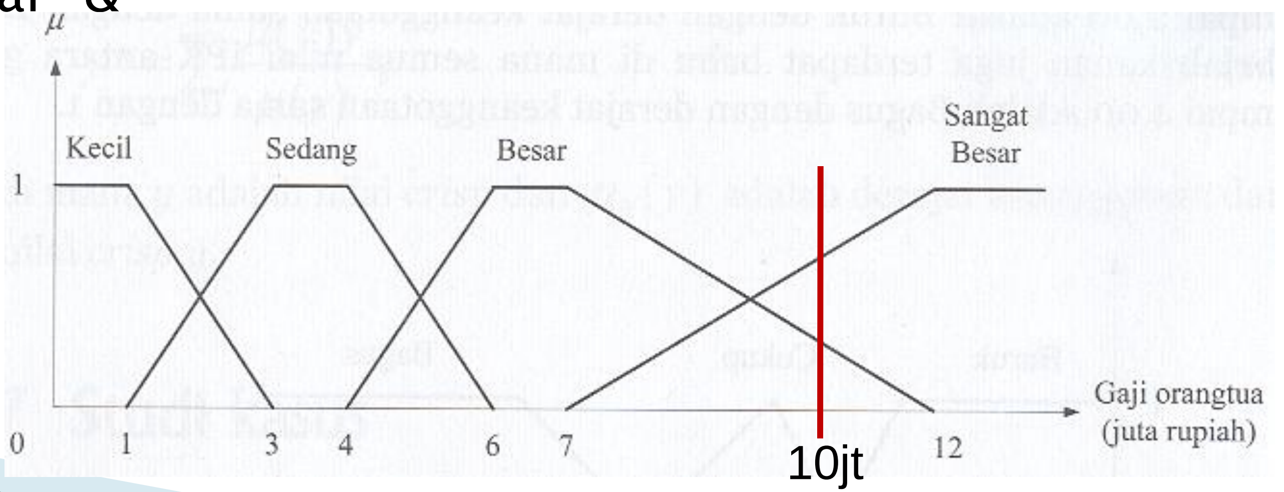
- Fungsi Keanggotaan Gaji:

Gaji 10jt: Nilai “Besar” &

“sangat besa

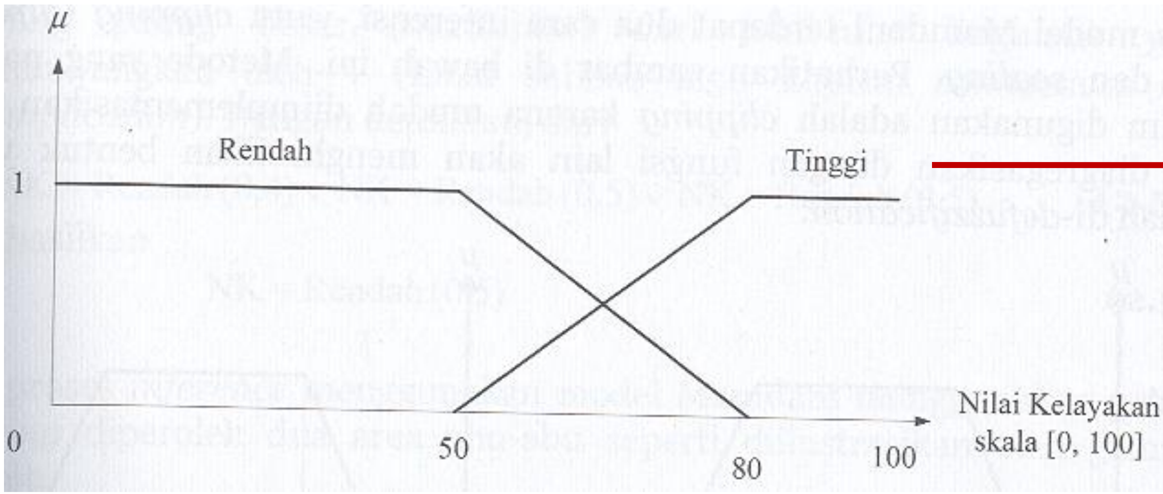
Besar: 0.4

Sangat besar: 0.6



2. Inference

- Misal nilai kelayakan di representasikan sebagai fungsi keanggotaan trapesium



- Misal aturan fuzzy berdasarkan pengetahuan pakar sbb:

Nilai Liguistik GAJI

Nilai Liguistik IPK

IPK \ Gaji	Kecil	Sedang	Besar	Sangat Besar
Buruk	Rendah	Rendah	Rendah	Rendah
Cukup	Tinggi	Rendah	Rendah	Rendah
Bagus	Tinggi	Tinggi	Tinggi	Rendah

2. Inference (cont'd)

- Dari tabel sebelumnya diperoleh 12 aturan fuzzy:

1. IF IPK = Buruk AND Gaji = Kecil THEN NK = Rendah
2. IF IPK = Buruk AND Gaji = Sedang THEN NK = Rendah
3. IF IPK = Buruk AND Gaji = Besar THEN NK = Rendah
4. IF IPK = Buruk AND Gaji = Sangat Besar THEN NK = Rendah
5. IF IPK = Cukup AND Gaji = Kecil THEN NK = Tinggi
6. IF IPK = Cukup AND Gaji = Sedang THEN NK = Rendah
7. IF IPK = Cukup AND Gaji = Besar THEN NK = Rendah
8. IF IPK = Cukup AND Gaji = Sangat Besar THEN NK = Rendah
9. IF IPK = Bagus AND Gaji = Kecil THEN NK = Tinggi
10. IF IPK = Bagus AND Gaji = Sedang THEN NK = Tinggi
11. IF IPK = Bagus AND Gaji = Besar THEN NK = Tinggi
12. IF IPK = Bagus AND Gaji = Sangat Besar THEN NK = Rendah

2. Inference (cont'd)

- Untuk mahasiswa A (IPK 3.00 dan Gaji 10jt):

Aturan yang memenuhi adalah aturan 7,8,11,12 yaitu:

IF IPK=cukup(0.5) AND Gaji=Besar(0.4) THEN NK=Rendah(0.4)

IF IPK=cukup(0.5) AND Gaji=Sangat Besar(0.6) THEN NK=Rendah(0.5)

IF IPK=Bagus(0.5) AND Gaji=Besar(0.4) THEN NK=Tinggi(0.4)

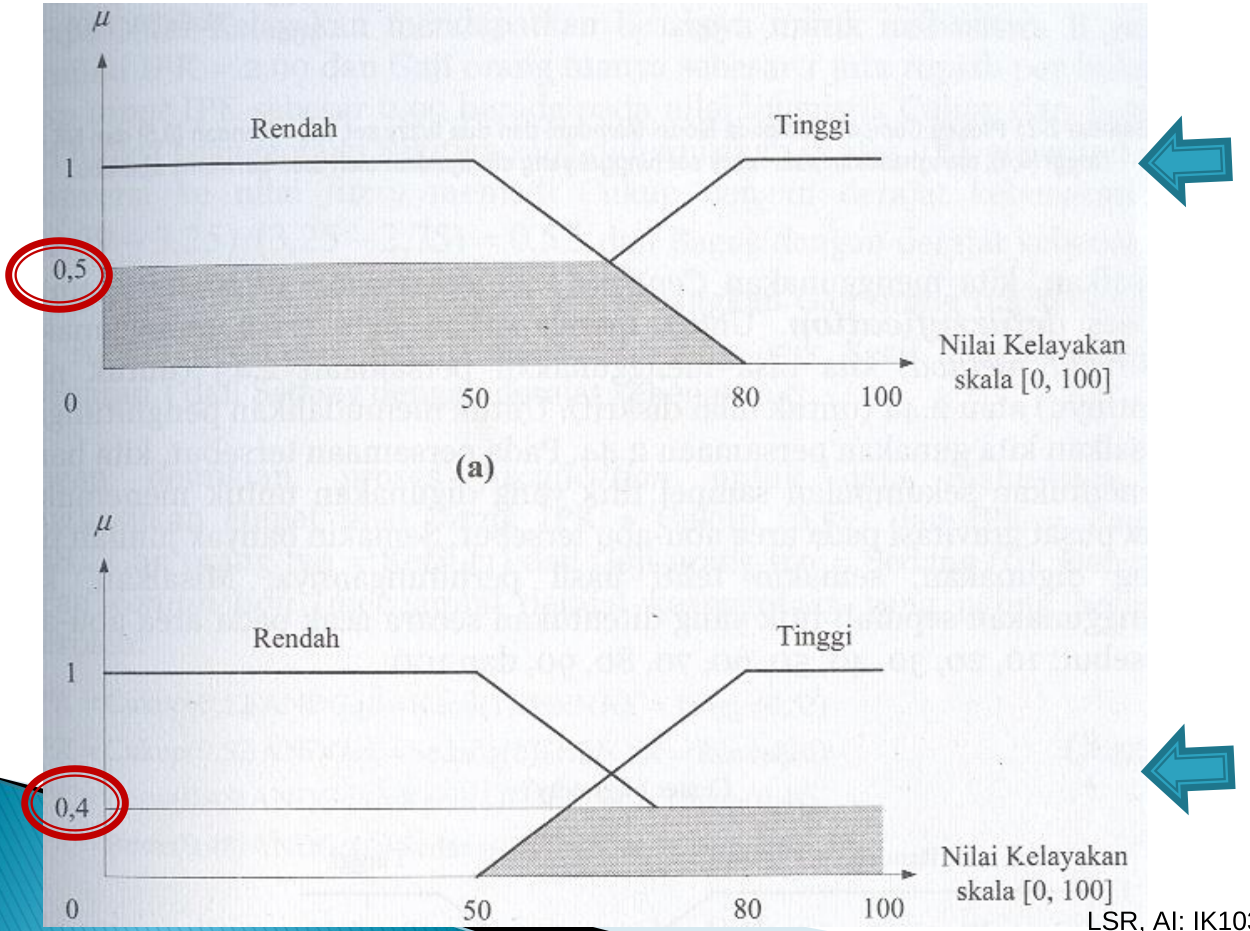
IF IPK=Bagus(0.5) AND Gaji=Sangat Besar(0.6) THEN NK=Rendah(0.5)

- Terdapat 1 dengan NK=Tinggi dan 3 dengan NK=Rendah; pilih derajat keanggotaan yg paling besar.
- Jadi diperoleh NK=Tinggi (0.4) dan Rendah(0.5).

Cat: derajat keanggotaan dipilih yg minimum antara IPK dan Gaji pada aturan inf.

2. Inference (cont'd)

- NK=Tinggi (0.4) dan Rendah(0.5)

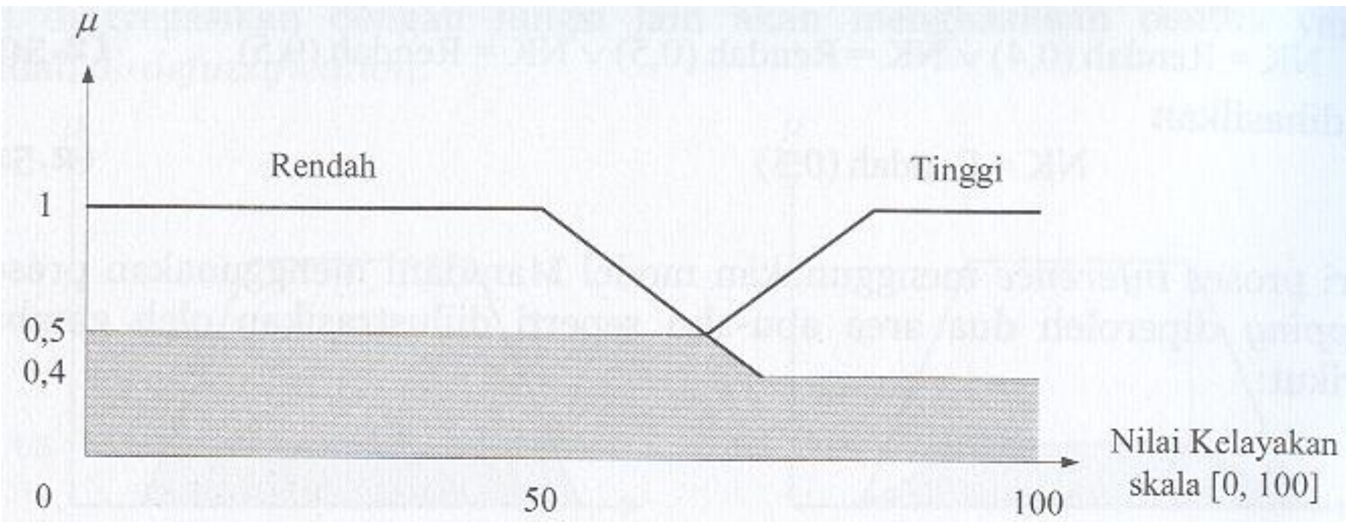


NK= Rendah(0.5)

NK= Tinggi(0.4)

3. Defuzzyfication

a. Proses composition: agregasi hasil clipping dari semua aturan fuzzy (NK=rendah(0.5) dan NK=Tinggi(0.4)).

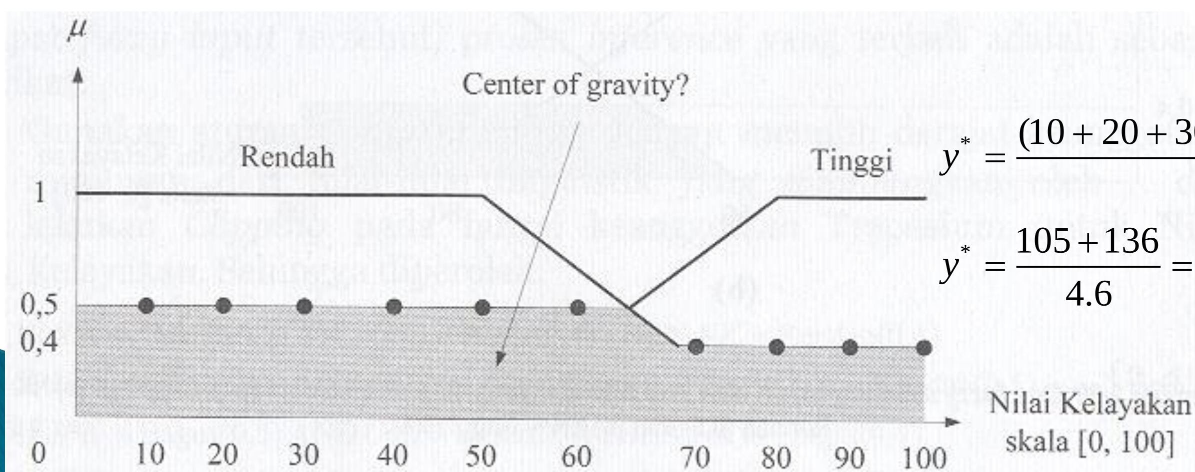
b. 

3. Defuzzyfication

- Centroid Method: tentukan sekumpulan sampel kemudian hitung titik pusat grafitasi.

$$y^* = \frac{\int y\mu_R(y)dy}{\int \mu_R(y)dy} \quad \textit{untuk masalah kontinu}$$

$$y^* = \frac{\sum y\mu_R(y)}{\sum \mu_R(y)} \quad \textit{untuk masalah diskrit}$$



$$y^* = \frac{(10+20+30+40+50+60)0.5+(70+80+90+100)0.4}{6(0.5)+4(0.4)}$$

$$y^* = \frac{105+136}{4.6} = 52.39$$

Kesimpulan

- Jadi untuk mahasiswa A (IPK 3.00 dan Gaji 10jt) mendapatkan nilai kelayakan sebesar = 52.39.
- Dengan cara yang sama untuk mahasiswa B (IPK 2.99 dan Gaji 1jt) mendapatkan nilai kelayakan sebesar = 69.66.
- Kesimpulannya: yang akan mendapatkan beasiswa dari 2 mahasiswa tsb adalah mahasiswa B.