On the calculation of pressure distribution in water distribution network

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Introduction

- The municipal water industry : to produce safe drinkable water and to provide excellent service to user. It is expected that year round water supply can't be interrupted
- Generally the water distribution system costs the greater part in municipal water construction project
- Suitable selection of pipe diameter , service reservoir capacity and location can save construction cost significantly
- Hence pipe network water pressure analysis play a very important role in municipal water engineering planning design work

Pipe network analysis

• Generally to simplify water pressure calculation we assume pipe network's water flow is steady flow, thus omit time factor in influencing flow volume.

Two important conditions to achieve water pressure balance in pipe network:

• Flow volume balance condition: for any two or more water pipe connection point, inflow water volume equal to outflow water volume.

• Water head balance condition: No matter which route the water flow through, the pressure difference between any two points in the network should be the same.

Pipe network water pressure calculation formula

- The Hazen-Williams flow equation is the most widely accepted and used for calculating pressure pipe flow conditions
- Given pressure drop (*lb/ft²*), the flow rate (*ft³/s*) can be calculated using (in U.S. Customary System)

$$Q_{ij} = 0.4329\, C_h\, D_{ij}^{2.63} \left(\frac{1}{L_{ij}} \left(\frac{p_i - p_j}{\gamma} + z_i - z_j \right) \right)^{0.54}$$

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Q_{ij} : volume flow rate in pipe segment from node i to node j $\left(ft^3/s\right)$

p_i : pressure at node i $\left(lb/ft^2\right)$

z_i : elevation at node i $\left(ft\right)$

L_{ij} : length of pipe segment from node i to node j $\left(ft\right)$

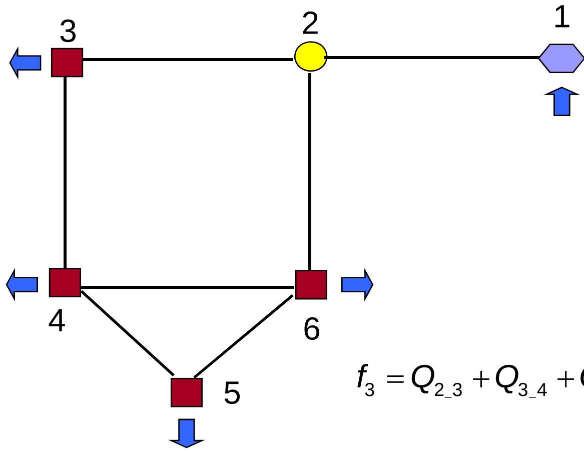
D_{ij} : inside diameter of pipe segment from node i to node j $\left(ft\right)$

γ : specific weight of water $\left(= 62.4\ lb/ft^3 \text{ at } 60^0 F\right)$

C_h : Hazen - Williams coefficient (dimensionless)

System Model

- Pipeline Network made up of connected pipeline to permit steady-state flow of water from one or more supply points to one or more delivery points
- The steady-state model is constructed by writing the continuity equation at each node in the system
- The water flow equation for each pipe connected to the node is then substituted to eliminate the pipe flow
- The result is a set of nonlinear simultaneous equations in pressures, which constitute the system model

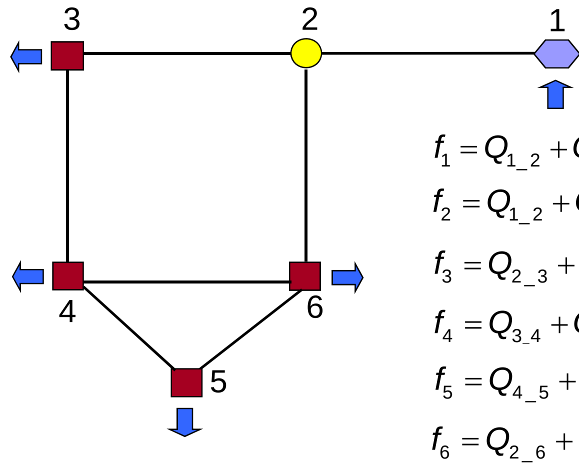


$$f_3 = Q_{2\_3} + Q_{3\_4} + QN_3 = 0$$

- Continuity equation at node *m*

$$f_m = Q_{j\_m} + Q_{m\_k} + QN_m = 0$$

QN_m is the node flow at node *m*



$$f_1 = Q_{1\_2} + QN_1 = 0$$

$$f_2 = Q_{1\_2} + Q_{2\_3} + Q_{2\_6} + QN_2 = 0$$

$$f_3 = Q_{2\_3} + Q_{3\_4} + QN_3 = 0$$

$$f_4 = Q_{3\_4} + Q_{4\_6} + Q_{4\_5} + QN_4 = 0$$

$$f_5 = Q_{4\_5} + Q_{5\_6} + QN_5 = 0$$

$$f_6 = Q_{2\_6} + Q_{4\_6} + Q_{5\_6} + QN_6 = 0$$

$$Q_{ij} = 0.4329\, C_h\, D_{ij}^{2.63} \left(\frac{1}{L_{ij}} \left(\frac{p_i - p_j}{\gamma} + z_i - z_j \right) \right)^{0.54}$$

Newton's method for solving systems of nonlinear equations

$$\mathbf{f}(\mathbf{x}) = \mathbf{0} \quad \text{with} \quad \mathbf{f} = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_N(\mathbf{x}) \right)^T$$

$$\text{and} \quad \mathbf{x} = \left(x_1, x_2, \ldots, x_N \right)^T$$

If $\mathbf{x}^{(0)}$ is an initial guess close to the true solution $\mathbf{x} = \mathbf{x}^*$,

$$\mathbf{f}(\mathbf{x}) \approx \mathbf{f}\left(\mathbf{x}^{(0)}\right) + J\left(\mathbf{x}^{(0)}\right)\left[\mathbf{x} - \mathbf{x}^{(0)}\right]$$

Solving for the 'root' of this linear equation,

$$\mathbf{x}^{(1)} = \mathbf{x}^{(0)} - J^{-1}\left(\mathbf{x}^{(0)}\right) \mathbf{f}\left(\mathbf{x}^{(0)}\right)$$

Repeating the above process, we obtain

$$\mathbf{x}^{(n+1)} = \mathbf{x}^{(n)} - J^{-1}\left(\mathbf{x}^{(n)}\right) \mathbf{f}\left(\mathbf{x}^{(n)}\right) \qquad n = 0, 1, 2, \ldots$$

$*$ Finding a good $\mathbf{x}^{(0)}$ usually is not an easy task

System of *n* non-linear equation in *n* variables and minimization of a function from R^n to R

$\mathbf{f}(\mathbf{x}) = 0$ where $\mathbf{f} = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_n(\mathbf{x}))^T$ and $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$

has a solution at $\mathbf{x} = (x_1, x_2, \cdots, x_n)^T$ when the function F defined by

$$F(x) = \frac{1}{1 + \|\mathbf{f}(\mathbf{x})\|}$$

where $\|\mathbf{f}(\mathbf{x})\| = \sqrt{f_1^2(\mathbf{x}) + f_2^2(\mathbf{x}) + \cdots + f_n^2(\mathbf{x})}$ has the maximum value 1

Genetic Algorithm

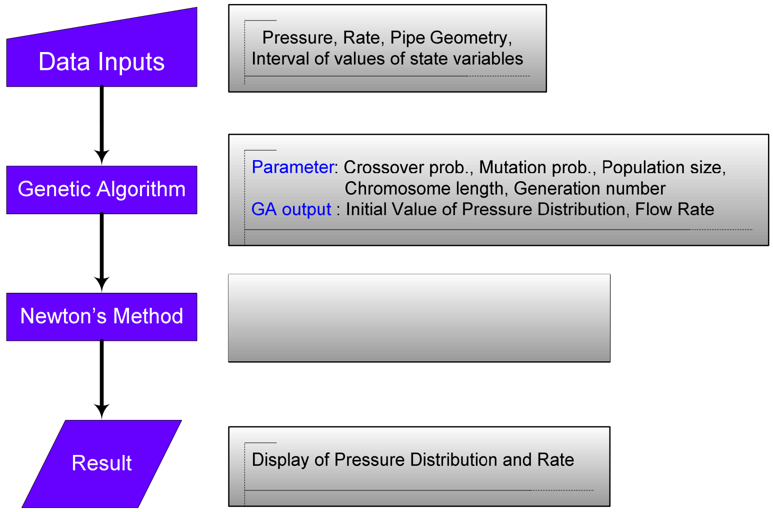
- To tackle the problem of choosing appropriate initial guess for Newton's method (for balancing problem)
- Balancing problem → Optimization problem

$\mathbf{f}(\mathbf{x}) = \mathbf{0}$ → *Fitness function*:

$$\max\ F(x) = \frac{1}{1 + \|\mathbf{f}(\mathbf{x})\|}$$

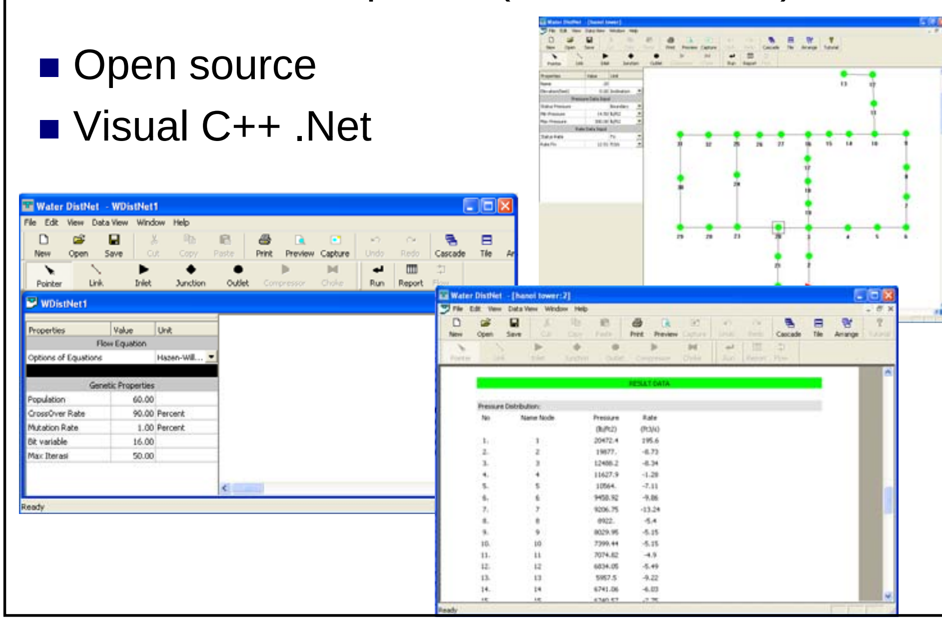
with $\|\mathbf{f}(\mathbf{x})\| = \sqrt{f_1^2(\mathbf{x}) + f_2^2(\mathbf{x}) + \cdots + f_N^2(\mathbf{x})}$

Process for solving the system model



Software development (Wdistnet v.1.0)

- Open source
- Visual C++ .Net



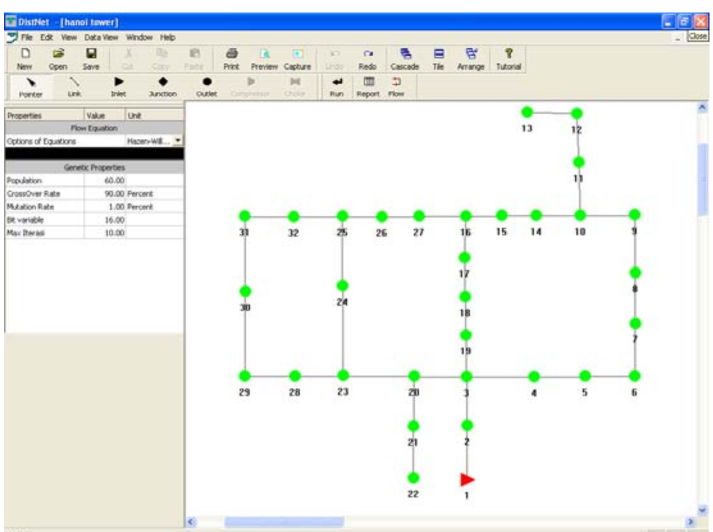
Case Study 1: Hanoi Network

- Consist of : 1 reservoir, 31 demand nodes,34 pipes,
- zero elevation
- input data (pressure on reservoir, diameter, length, and flow rate on each node):

  Table of node input data

  Table of pipe input data
- output: pressure on each node, flow rate on each pipe, and flow direction.

Hanoi Network



Results: Pressure distribution

Table of Head/Pressure Distribution				
		Head/Pressure		Rate
	Name	Pressure		
No	Node	(lb/ft2)	Head(m)	(ft3/s)
1	1	20472.4	99.9998	195.6
2	2	19876.9	97.09101	-8.73
3	3	12488.2	61.00005	-8.34
4	4	11627.8	56.79733	-1.28
5	5	10564	51.60108	-7.11
6	6	9458.89	46.20304	-9.86
7	7	9206.73	44.97134	-13.24
8	8	8921.98	43.58044	-5.4
9	9	8029.92	39.22307	-5.15
10	10	7399.41	36.14327	-5.15
11	11	7074.79	34.55763	-4.9
12	12	6834.02	33.38156	-5.49
13	13	5957.47	29.09995	-9.22
14	14	6741.02	32.92729	-6.03
15	15	6740.58	32.92514	-2.75

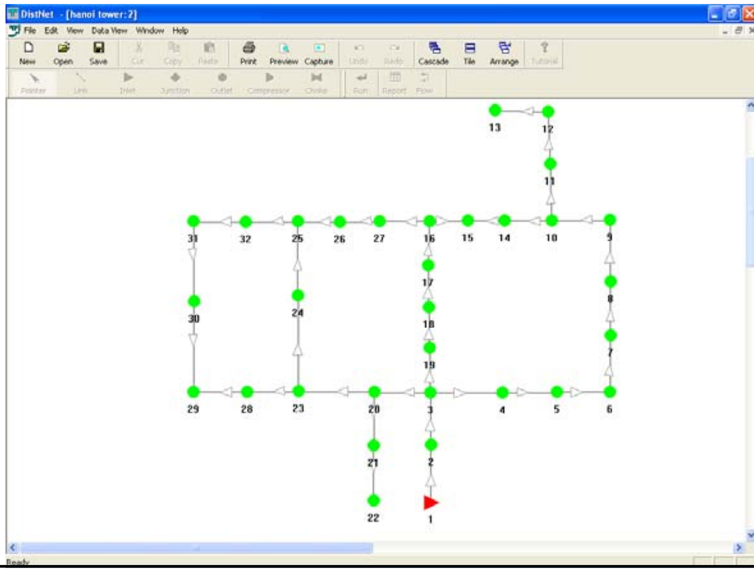
Table of Head/Pressure Distribution				
		Head/Pressure		Rate
	Name	Pressure		
No	Node	(lb/ft2)	Head(m)	(ft3/s)
16	16	7168.13	35.01356	-3.04
17	17	9773.95	47.74199	-8.49
18	18	10890.9	53.19786	-13.19
19	19	11946.3	58.35308	-0.59
20	20	10811.9	52.81197	-12.51
21	21	8864.63	43.30031	-9.12
22	22	7787.94	38.04109	-4.76
23	23	7722.15	37.71973	-10.25
24	24	7217.51	35.25476	-8.04
25	25	6548.29	31.98588	-1.67
26	26	6607.12	32.27324	-8.83
27	27	6802.88	33.22945	-3.63
28	28	6443.09	31.47202	-2.84
29	29	5901.46	28.82636	-3.53
30	30	5912.97	28.88258	-3.53
31	31	5964.1	29.13233	-1.03
32	32	6119.11	29.8895	-7.9

Results: Rate Distribution

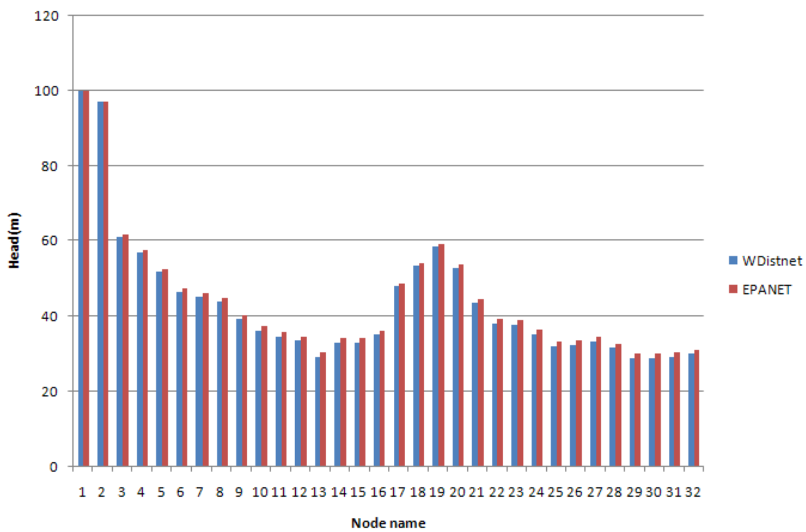
Table of Rate Distribution:						
No	Segment			Diameter	Length	Flow Rate
	Link Name	From Node	To Node	(ft)	(ft)	(ft3/s)
1	Link - 33	2	1	3.32	328	195.6
2	Link - 34	3	2	3.32	4429.1	186.87
3	Link - 35	3	4	3.32	2952.8	72.834
4	Link - 36	4	5	3.32	3773	71.554
5	Link - 37	5	6	3.32	4757.2	64.444
6	Link - 38	6	7	3.32	1476.4	54.584
7	Link - 39	7	8	3.32	2788.7	41.344
8	Link - 40	8	9	2.49	2788.7	35.944
9	Link - 41	9	10	2.49	2624.7	30.794
10	Link - 42	10	11	2.49	3116.8	19.61
11	Link - 43	11	12	2.49	3937	14.71
12	Link - 44	12	13	1.992	11483	9.22
13	Link - 45	10	14	1.328	2624.7	6.0342
14	Link - 46	14	15	0.996	1640.4	7.05E-02
15	Link - 47	15	16	0.996	1804.5	2.7458

Table of Rate Distribution:						
No	Segment			Diameter	Length	Flow Rate
	Link Name	From Node	To Node	(ft)	(ft)	(ft3/s)
16	Link - 48	16	17	1.992	8956.7	18.989
17	Link - 49	17	18	2.49	5741.5	27.479
18	Link - 50	18	19	2.49	2624.7	40.669
19	Link - 51	19	3	2.49	1312.3	41.259
20	Link - 52	16	27	1.992	2460.6	13.204
21	Link - 53	27	26	1.66	984.25	9.5737
22	Link - 54	26	25	0.996	2788.7	0.7437
23	Link - 55	25	24	1.992	4265.1	13.605
24	Link - 56	24	23	2.49	4035.4	21.645
25	Link - 57	23	20	2.49	8694.2	38.046
26	Link - 58	20	3	3.32	7217.9	64.436
27	Link - 59	25	32	1.992	3116.8	12.679
28	Link - 60	32	31	1.66	2821.5	4.779
29	Link - 61	31	30	1.328	492.13	3.749
30	Link - 62	30	29	0.996	5249.3	0.21904
31	Link - 63	29	28	1.328	6561.7	3.311
32	Link - 64	28	23	1.328	4921.3	6.151
33	Link - 65	20	21	1.66	4921.3	13.88
34	Link - 66	21	22	0.996	1640.4	4.76

Results: Water flow Direction



Comparation Wdistnet vs EPANET 2.0



Case Study 2 : Badak Singa (Bandung) Network

Consist of

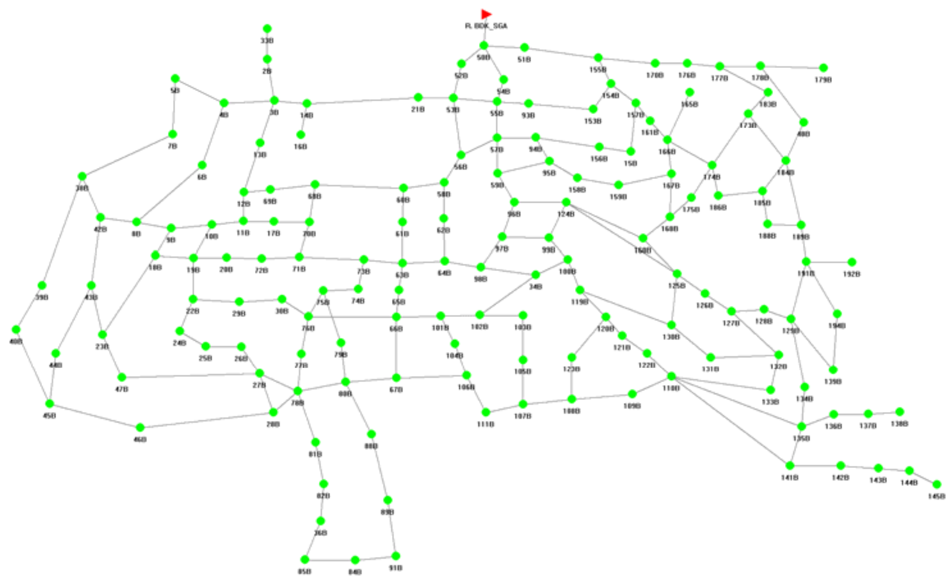
- 1 reservoir (Badak Singa reservoir),
- 155 demand nodes
- 202 pipes

Input Data of Badak Singa (Bandung) Distribution

- The data required :
  - Node properties (Head/pressure at Reservoir Badak Singa, rate at all demand point) Click to show Node Properties
  - Pipeline properties (length, diameter)

    Click to show Pipeline Properties
  - elevation data on each node.

Schematic of water pipeline network of Badak Singa



Result of WDistnet: Pressure Distribution

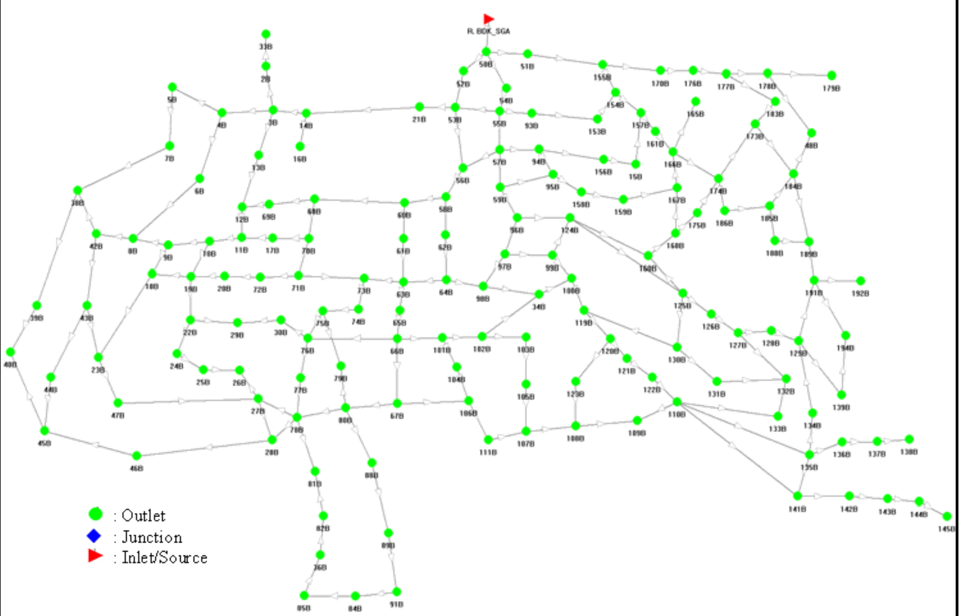
Head Distribution:

No	Name Node	Head(m)	Rate(LPS)
1	52B	743.684	-2.68
2	54B	743.804	-3.69
3	53B	743.246	-1.1
4	21B	743.009	-1.1
5	55B	743.255	-1.1
6	93B	742.628	-1.1
7	57B	743.033	-1.1
8	94B	742.248	-1.1
9	56B	743	-1.1
10	58B	742.38	-3.79
11	60B	742.252	-9.83
12	61B	741.676	-12.84
13	62B	741.738	-5.28
14	63B	741.031	-11.98
15	64B	741.107	-6.66
16	73B	740.809	-11.87
17	74B	740.637	-27.15
18	65B	740.667	-22.52
19	59B	742.625	-3.79
20	95B	740.573	-8.45

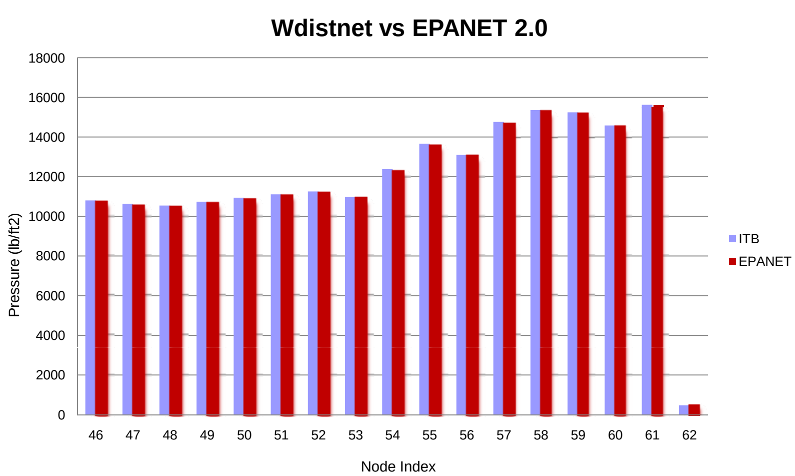
Click to show the complete result

Compare with EPANET Software :
Click

Water Flow Direction



Comparation



End of Presentation