Implementation of Genetic Algorithm to Improve Convergence of Newton's Method in Predicting Pressure Distribution in a Complex Gas Pipeline Network System. Case Study: Off-take Station, ST-WLHR Indonesia

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Abstract

Natural gas pipeline network was made up of several points of supply and several delivery points connected by pipeline, allowing steady-state flow of natural gas in the complex pipeline network system. The system model consists of a set of nonlinear simultaneous equations obtained by writing the continuity equation at each node in the system. Therefore an iterative technique, for example Newton's method, can be applied to obtain a solution of these systems of equations. The method requires that a good initial guess be given for all unknowns for convergence of the method. Usually this is not an easy task. In this paper we propose a technique that uses a Genetic Algorithm to overcome the problem. This method is basically used to generate initial pressure and rate data of the model, which will be executed by Newton's method. The model has been validated with data from ST-WLHR station which is an off-take station that distributes gas to several industrial consumers in Jakarta-Indonesia. This distribution system consists of a complex pipeline network.

Keywords: *genetic algorithm, steady-state gas pressure distribution, complex pipeline network*

1. Introduction

Recently, natural gas has an important role in providing clean and environmental friendly energy toward our community. Hence, gas demand as energy source is increasing despite of its relatively higher price than oil. Natural gas is widely used as a source of energy for industrial needs as well as public consumption.

Gas operator companies have the responsibility to provide gas to the consumers with certain rate and pressure described in the sales contracts. Therefore, the companies should be able to preserve gas pressure distribution and rate in every delivery point (customer's entry point) to fulfill the contract, as well as predicting the consumer's increasing demand of gas in the future.

This paper is mainly focused on determining gas pressure distribution of a complex pipeline network. Natural gas pipeline network was considered to have been made up of connected pipeline so as to permit the steady - state flow of gas from one or more points of supply to one or more points of delivery.

The system model being used to represent the natural gas pipeline system is the balancing-of-nodes flow solution method. Stoner [6] first made the method practical for large problems. He proposed a steady-state model by writing the continuity equation at each node in the system. The gas flow equation for each pipe connected to the node is then substituted to eliminate the pipe flow. This results in a set of nonlinear simultaneous equations, which constitute the system model.
Determining gas pressure distribution in the natural gas pipelines is similar to obtaining a solution of this system of nonlinear equations. This system is usually solved by an iterative technique such as Newton's method, which is well - known as the most effective iterative method. However, an iterative technique requires that initial estimates be given for all unknowns in the system of equation. Even for small system, very good initial guesses were required for convergence of the method.

Therefore finding initial estimates for all unknowns, especially for large system, is not an easy task. In order to overcome the problem, this study has proposed a technique that uses a Genetic Algorithm [1], [7] in order to find initial estimates for all unknowns. Instead of supplying initial estimates for each unknown, the proposed technique only requires an interval of values for each unknown to be considered. The values obtained by Genetic Algorithm can then be used as initial guesses

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for the iterative technique.

2. System Model

Generally, models have been written based on assumption of horizontal pipeline. However, in actual condition, pipelines have been laid according to the geographical contour. In this paper, model has been written for the horizontal pipeline. From this horizontal flow model, we derived the non-horizontal one, which is more applicable in practice [5].

As mentioned before, the system model used to represent the natural gas pipeline system is the balancing of nodes flow solution method. This method requires an existing nodes and node – connecting elements (NCE). In this paper, a gas pipeline network is considered as the nodes and NCE. This network consists of a number of pipes, of specified diameters and lengths, which are joined at N nodes, numbered $i = 1,2,\cdots,N$. The pressure is specified at some of these nodes. Pipelines NCE affect the gas flow rate through the element by pressure decrease and unaltered gas flow rate. Nodes represent points where one or more NCE terminate and where flow enters or leaves the system. Nodes are also the reference points for system pressures.

Here the system model is built based on the following assumptions: horizontal pipelines, isothermal condition, steady-state flow, compressors and control valves are not included.

As mentioned before, the type of NCE considered in this paper is restricted to the pipelines. A pipeline connects two nodes: i and j, having a length of L_{ij} (in miles) and inside diameter D_{ij} (in inch). As pipeline model equation we use Panhandle A formula for horizontal flow [8]:

$$Q_{ij} = S_{ij} \frac{CED_{ij}^{2.6128}(\left|p_i^2 - p_j^2\right|)^{0.5394}}{SGg^{0.4606} T^{0.5394} L_{ij}^{0.5394}} \qquad (1)$$

in which Q_{ij} is the gas flow rate in the pipe section between nodes i and j, p_i and p_j are pressures at nodes i and j, and C is a constant. The subscript i denotes the "from" node and the subscript j denotes the "to" node. Flow from i to j is positive. Since in some pipes the flow is bi-directional, the term $S_{ij} = (p_i - p_j)/\left|p_i - p_j\right| = \pm 1$ is introduced to provide the proper flow sign. Flows are in MMscfd and pressures are in psia. We shall assume for purpose of simplifications that all pipeline segments experience the following conditions: T = 60 F, SGg = 0.6, and E = 0.92. Hence we have simplified form of the Panhandle A flow equation expressed by:

$$Q_{ij} = S_{ij} \frac{KD_{ij}^{2.6182}(\left|p_i^2 - p_j^2\right|)^{0.5394}}{L_{ij}^{0.5394}} \qquad \text{(with K= 8.2634*10}^{-4}\text{)} \qquad (2)$$

In this paper, we use Panhandle A equation to model gas flow in pipeline. However, Panhandle B and Weymouth equation is also applicable in this situation. These three equations have the same role. The Weymouth, Panhandle A, and Panhandle B equations were developed to simulate compressible gas flow in long pipelines. Those equations were developed from the fundamental energy equation for compressible flow, but each has a special representation of the friction factor to allow the equations to be solved analytically [2].

In order to develop the system model, the continuity relationship for each node in the system is written. This equation relates the flow in the pipe attached to that node and the node flow. Using the analogy of Kirchoff's current law in electricity for electrical network to gas flow in pipeline network, we obtain that the algebraic sum of gas flow entering and leaving any node is zero. Thus we have the continuity equation of node as:

$$f_m = Q_{j\_m} + Q_{m\_k} + QN_m = 0 \qquad (3)$$

Subscript in the above equation represents nodal number. Subscripts j and k are neighbouring nodes of node m, whereas j_m and m_k represents pipe sections between nodes j and m, and nodes m and k respectively. QN is the node flow of node m and Q_{j_m} is the gas flow rate in j_m pipe section. Flows into the node are considered to be positive. Herein f represents the flow imbalance at the node and will be zero in case the system is in balance. Since we have N nodes in the network, there are N continuity equations similar to the above equation. Figure 2 shows a 24 nodal system in which gas is transported from one supply point (node 1) to the 8 delivery points (nodes 4, 6, 9, 11, 13, 17, 21, 24). For node 3 its continuity equation is:

$$f_3 = Q_{2\_3} + Q_{3\_4} + Q_{3\_5} = 0 \qquad (4)$$

Substituting equation (2) into equation (4) we obtain

$$\begin{aligned} f_3 = S_{2\_3} & \frac{K\,D_{2\_3}^{2.6182}(\left|p_2^2 - p_3^2\right|)^{0.5394}}{L_{2\_3}^{0.5394}} + \\ S_{3\_4} & \frac{K\,D_{3\_4}^{2.6182}(\left|p_3^2 - p_4^2\right|)^{0.5394}}{L_{3\_4}^{0.5394}} + \\ S_{3\_5} & \frac{K\,D_{3\_5}^{2.6182}(\left|p_3^2 - p_5^2\right|)^{0.5394}}{L_{3\_5}^{0.5394}} = 0 \end{aligned} \qquad (5)$$

Similarly the node continuity equations for the other nodes in the system of Figure 2 are formulated. Since there are 24 nodes in the system of Figure 2, there are 24 continuity equations similar to and including equation (5). This system of nonlinear equations represents the steady-state model for the gas pipeline system of Figure 2. When the values of all the variables in this system of equations are such that the $f_i\,(i = 1,2,\cdots,24)$'s are reduced to or near zero, the model is said to be balance. For each node there is one pressure p and one node out (in) flow QN. For each pipe segment, there is one parameter, the pipe diameter, which is considered as constant in this paper. Hence for the system in Figure 2, there will be a total of 48 variables. Since we have 24 nodes, it should then be possible to use the 24 nodal equations to solve for 24 of the 48 variables. The 24 variables to be determined are called unknowns or *state variables*. The other variables, called *decision variables*, must be given fixed values so the system of equations can be solved. Since only 23 of the equations are independent in QN, at least one QN must be a state variable. It is also necessary that at least one pressure be specified so that a pressure reference is given for the system.

In a general system consisting of N nodes and M node connecting elements (NCE), there will be a total of $2N + M$ variables. Since we have N nodes, then it should be possible to use the N nodal equations to solve for N of the $2N+M$ variables. The N variables to be determined are called unknowns or *state variables*. The other $N+M$ variables, called *decision variables*, must be given fixed values so the system of equations can be solved. Since only $N - 1$ of the equations are independent in QN, at least one QN must be a state variable. It is also necessary that at least one pressure be specified so that a pressure reference is given for the system. The N nodal equations are nonlinear simultaneously equations, therefore an iterative technique can be applied to obtain a solution. In this paper, we used the effective iterative techniques i.e. Newton's method for solving the nonlinear system. This solution will make the system balance. However, for convergence of the iterative technique, a good initial guess for all unknowns must be given. Usually this is not an easy task. To overcome the problem, in this work we propose a technique that use a Genetic Algorithm to find an initial guess for all unknowns and then applying an iterative technique, Newton's method, to find a solution of the corresponding system of nonlinear simultaneously equations. Rather than supply initial guess for each unknown, the proposed technique only requires an interval of values for each unknown to be considered. In the following section, we will briefly describe both the Genetic Algorithm, which will provide an initial guess for Newton's method, and the Newton's method itself.

3. Solution of the System Model

As mentioned in section 2, the N nodal equations of the system model are nonlinear simultaneous equations. Therefore an iterative technique can be used to obtain a solution. In this work, we used Newton's method as mentioned before. However the iterative method needs a good initial guess for convergence of the methods. One robust technique that may result a good initial value is Genetic Algorithm that have been known as a good technique for finding optima. Figure 1 describes process to solve the system model.

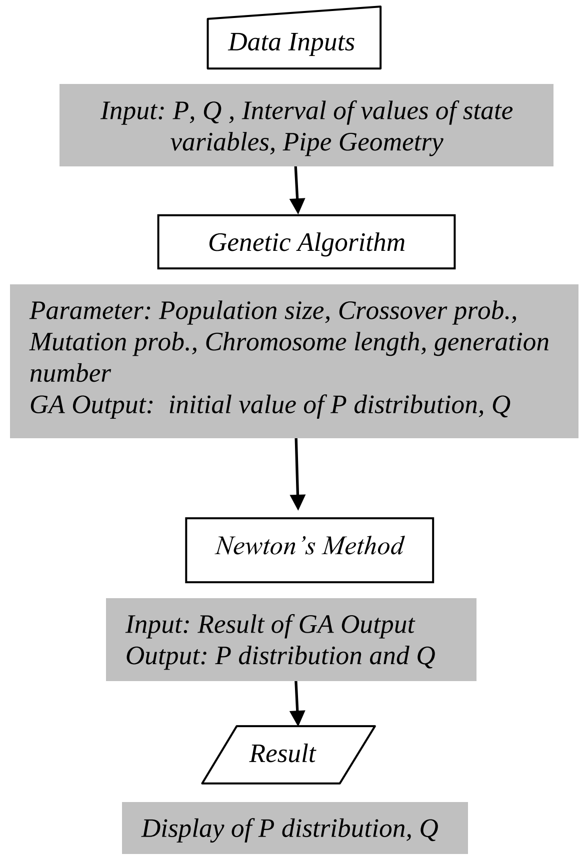

Figure 1. Process to solve the system model

3.1 Genetic Algorithm

Iterative method may solve a nonlinear system equation effectively. However, the method may not converge unless the initial approximation is a good one. But finding a good initial guess to the root of the system of nonlinear equations usually is not an easy task. To overcome the problem we will use a Genetic Algorithm for finding an initial approximation to the root.

Genetic Algorithm is a class of algorithms inspired by evolution. These algorithms encode solution to a specific problem on a chromosome like data structure and apply recombination operators to these structures so as to preserve critical information. An implementation of a Genetic Algorithm begins with a population of random chromosome. The members of this population are usually strings of zeros and ones. Each string in the population corresponds to a chromosome and each binary element of the string to a gene. Instead of binary coding, one can also consider a real coding. This initial population is generated randomly. The chromosomes are then evaluated and allocated reproductive opportunities in such a way that those which represent a better solution to the problem are given more chances to reproduce. Basic steps in a Genetic Algorithm are [9]:

1. Generate randomly an initial population of chromosomes.
2. Calculate the fitness, defined according to some specified criteria, of all the members of the population and select individuals for the reproduction process. The fittest are given a greater probability of reproducing in proportion to the value of their fitness.
3. Apply the genetic operators of crossover and mutation to the selected individuals to create new individuals and thus a new generation. Crossover exchanges some of the bits (genes) of the two chromosomes, whereas mutation inverts any bit(s) of the chromosome depending on a probability of mutation. Thus a 0 may be changed to a 1 or vice versa.

Then again step 2 is followed until the condition for ending the algorithm is reached.

Traditionally Genetic Algorithm have been used for optimization problems. Among the advantages of applying Genetic Algorithm to optimization problems is that Genetic Algorithm do not have much mathematical requirements about the optimization problems. Due to their evolutionary nature, Genetic Algorithm will search for solution without regard to the specific inner workings of the problem. Genetic Algorithm can handle any kind of objective functions and any kind of constraints (i.e. linear or nonlinear) defined on discrete or continuous search spaces [7]. Genetic Algorithm can also be used to find the roots of transcendental equations by first converting the problem into an optimization problem [9]. Inspired by the recent work of Agarwal [9] we can convert the problem of finding the roots of system of nonlinear equations into a multi-modal optimization problem and then used a Genetic Algorithm to solve the optimization problem and hence obtain the roots of the corresponding system nonlinear equations [3]. In this paper we use a simple Genetic Algorithm to find an approximate solution of the system of nonlinear equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ discussed above. We have used binary encoding for the problem, and the fitness function chosen was

$$F(\mathbf{x}) = norm(\mathbf{f}(\mathbf{x})) = \|\mathbf{f}(\mathbf{x})\| \qquad (13)$$

in which

$$\|\mathbf{f}(\mathbf{x})\| = \sqrt{f_1^2(\mathbf{x}) + f_2^2(\mathbf{x}) + \cdots + f_N^2(\mathbf{x})} \qquad (14)$$

is the Euclidean norm of $\mathbf{f}(\mathbf{x})$. Hence at the vicinity of the root of $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ we expect that the value of the fitness function will be close to 0 and will be large positive number in case $\mathbf{x}$ is far from the root. Thus our problem, i.e. finding a solution for $\mathbf{f(x)} = \mathbf{0}$, initially will be formulated as a problem of finding $\mathbf{x}$ that minimize the function $F(\mathbf{x})$. The result of minimization problem of $F(\mathbf{x})$ using simple Genetic Algorithm will then be used as an initial guess of Newton's method to the equation $\mathbf{f(x)} = \mathbf{0}$. This is because we observed that in some problems, the convergence of Genetic Algorithm is rather slow and their accuracy is lower than the accuracy of the solution found by conventional method. Hence we propose to use Genetic Algorithm combined with the Newton's method. See also [4].

3.2 Newton's Method for Solving Nonlinear System

In this paper we used Newton's method for solving nonlinear system. The Newton's method has been known as an effective iterative method. But, its convergence depends on initial guess. If the initial guess is not a good one, the method may diverge.

Let us denote a system of N nonlinear equations as $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ where $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_N(\mathbf{x}))^T$ and $\mathbf{x} = (x_1, x_2, \ldots, x_N)^T$.

Also denote the Jacobian matrix of $\mathbf{f}$ by J, that is, the matrix of all first derivatives with $J_{ij} = \partial f_i / \partial x_j$.

In the vicinity of any point $\mathbf{x}^{(0)}$, the Taylor theorem gives a linear approximation to $\mathbf{f}(\mathbf{x})$. If this point $\mathbf{x}^{(0)}$ is an initial guess close to the true solution $\mathbf{x} = \mathbf{x}^*$, hoping that the linear approximation is close to $\mathbf{f}(\mathbf{x})$, we may write

$$\mathbf{f}(\mathbf{x}) \approx \mathbf{f}(\mathbf{x}^{(0)}) + J(\mathbf{x}^{(0)})\left[\mathbf{x} - \mathbf{x}^{(0)}\right] \qquad (15)$$

in which the Jacobian matrix J is evaluated at $\mathbf{x} = \mathbf{x}^{(0)}$. Solving for the 'root' of this linear equation, giving us

$$\mathbf{x}^{(1)} = \mathbf{x}^{(0)} - J^{-1}(\mathbf{x}^{(0)})\,\mathbf{f}(\mathbf{x}^{(0)}) \qquad (16)$$

Repeating the above process, we obtain the Newton's method

$$\mathbf{x}^{(n+1)} = \mathbf{x}^{(n)} - J^{-1}(\mathbf{x}^{(n)})\,\mathbf{f}(\mathbf{x}^{(n)}) \qquad n = 0,1,2,\cdots \qquad (17)$$

in which the Jacobian matrix J is evaluated at $\mathbf{x} = \mathbf{x}^{(n)}$. The iteration formula can be written as $\mathbf{x}^{(n+1)} = \mathbf{x}^{(n)} + \mathbf{d}^{(n)}$ where $\mathbf{d}^{(n)}$ is the solution of the system of linear equation:

$$J(\mathbf{x}^{(n)})\,\mathbf{d}^{(n)} = -\mathbf{f}(\mathbf{x}^{(n)}) \qquad (18)$$

4. Implementation

Figure 2 shows a twenty four-node pipeline system delivering gas from one supply point (node 1) to eight delivery points (nodes 4, 6, 9, 11, 13, 17, 21 and 24). All data used in the model such as inside diameter and length of pipes, source pressure at node 1, amount of gas demand at nodes 4, 6, 9, 11, 13, 17, 21, and 24 are given. The problem is to find pressure distribution at nodes other than node 1 and the amount of gas that must be supplied at node 1 such that the system is in steady-state condition. Using data shown in table 1, Genetic Algorithm will carry out process to provide initial guess for Newton's method. Table 2 shows the result of Newton's method.

5. Conclusion

Simple Genetic Algorithm combined with Newton's method can be helpful in finding a solution of system of nonlinear simultaneous equation resulting from the nodal continuity equation of steady-state gas pipeline network. In this way simple Genetic Algorithm will generate an initial, hopefully a good one, guess of a solution obtained with Newton's method.

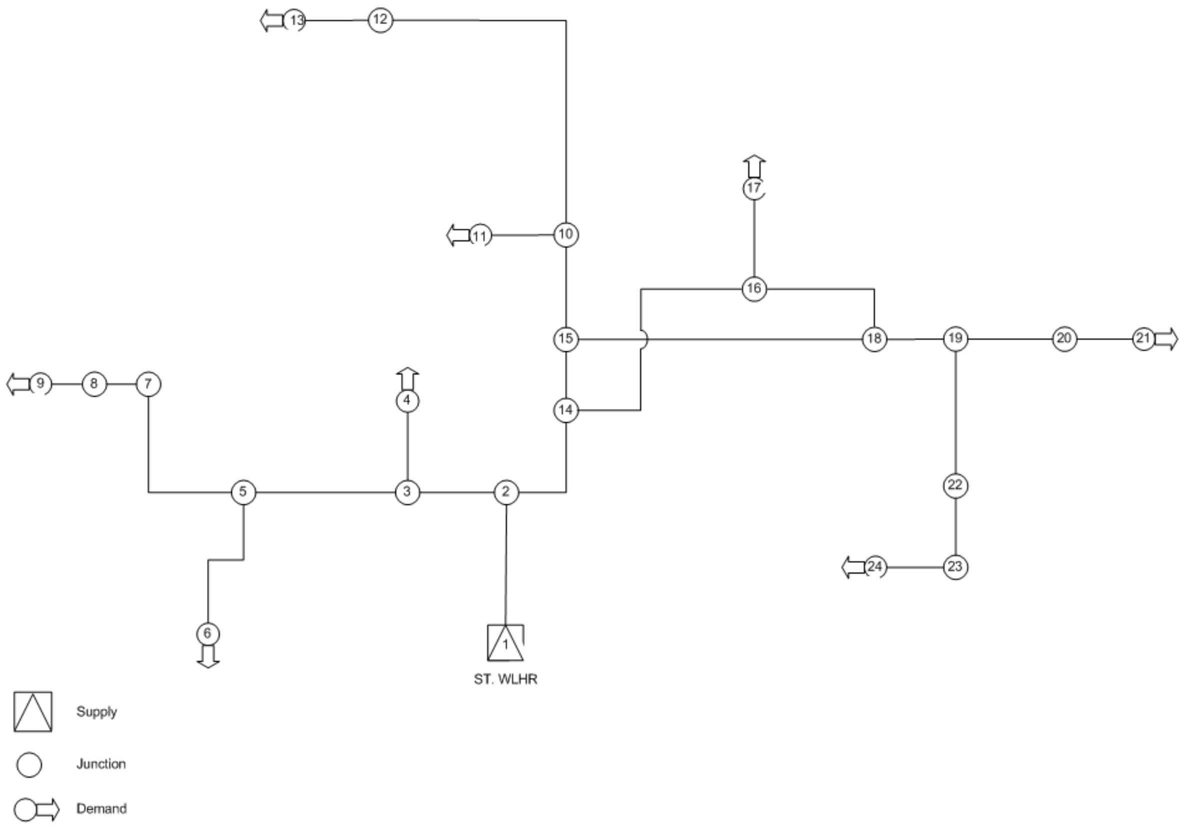

Figure 2. Schematic Gas Distribution Pipeline Network, Off – Take Station ST-WLHR

Table 1. User Input Data

Node Properties			Segment Properties			
Node	Pressure (psia)	Field Rate (MMscfd)	Link From	Link To	Pipe Length (miles)	Diameter (inches)
1	246.761	Unknown	1	2	0.29375	16
2	Unknown	0	2	3	0.36062	16
3	Unknown	0	3	4	0.03125	7.981
4	Unknown	-2.9372	3	5	0.35375	16
5	Unknown	0	5	6	1.375	6.065
6	Unknown	-3.0786	5	7	1.1038	16
7	Unknown	0	7	8	0.52812	10.02
8	Unknown	0	8	9	0.0375	7.981
9	Unknown	-3.9443	2	14	1.71	16

10	Unknown	0	10	11	0.08937	7.981
11	Unknown	-10.0684	10	12	4.4813	16
12	Unknown	0	12	13	0.03125	7.981
13	Unknown	-6.8464	14	15	0.311	16
14	Unknown	0	15	10	0.311	16
15	Unknown	0	14	16	3.15	6.065
16	Unknown	0	16	17	0.155	4.026
17	Unknown	-3.0419	16	18	0.391	6.065
18	Unknown	0	18	19	0.435	16
19	Unknown	0	19	20	0.684	6.065
20	Unknown	0	20	21	0.0932	4.026
21	Unknown	-0.0067	19	22	7.15	16
22	Unknown	0	22	23	0.621	12
23	Unknown	0	15	18	3.54	16
24	Unknown	-2.7569	23	24	0.0311	6.065

Table 2. Output Data

Node	**Pressure (psia)**	**Field Rate (MMscfd)**	**Node**	**Pressure (psia)**	**Field Rate (MMscfd)**	**Node**	**Pressure (psia)**	**Field Rate (MMscfd)**
1	246.761	32.6804	9	246.1373	-3.9443	17	242.7158	-3.0419
2	246.3945	0	10	245.0046	0	18	244.9907	0
3	246.3447	0	11	244.634	-10.0684	19	244.9851	0
4	246.3315	-2.9372	12	244.6936	0	20	244.9851	0
5	246.3191	0	13	244.6302	-6.8464	21	244.9851	-0.0067
6	243.9177	-3.0786	14	245.3034	0	22	244.8932	0
7	246.2917	0	15	245.1198	0	23	244.8609	0
8	246.1646	0	16	244.6612	0	24	244.8166	-2.7569

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