METODE KERJA VIRTUIL (*Virtual Work Method)*

Metode ini dapat digunakan untuk menyelesaikan struktur yang cukup kompleks, dengan cara ini kita meninjau keseimbangan energi dari struktur tersebut ketika mengalami mekanisme runtuhnya. Pada prinsipnya saat runtuh atau terjadi mekanisme, struktur akan mengalami deformasi-deformasi sehingga terjadi ; usaha luar $\Sigma W.\delta$ dan usaha dalam $\sum$Mp. θ

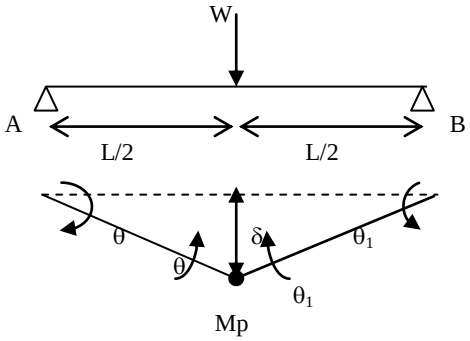
Kondisi keseimbangan menghendaki kerja/ usaha luar sama dengan kerja/ usaha dalam , yaitu $\Sigma W.\delta = \sum$Mp. θ

Metode ini berdasarkan teorema batas atas (Dalil kinematis), beban runtuh yang dihasilkan akan sama ataupun lebih besar dari nilai sebenarnya.

Prosedur perhitungannya:

1. tentukan lokasi sendi plastis yang mungkin terjadi, yaitu pada tempat beban titik, titik-titik perletakan, titik sambungan engsel, titik pada gaya gesernya nol (pada beban merata), dan titik dimana terdapat momen lentur maksimum.
2. pilih kemungkinan mekanisme bebas (*independent mechanism*) dan mekanisme gabungan *(composite mechanism)*
3. selesaikan persamaan kesetimbangan dengan prinsip kerja virtual dan tentukan beban runtuhnya
4. Periksa apakan M ≤ Mp pada semua tampang

Contoh : Balok sederhana (beban ditengah)



$\text{Tan } \theta = \frac{\delta}{L/2}$, θ = Kecil

$\theta = \frac{\delta}{L/2}$; $\delta = \frac{1}{2} . \theta . L$; $\theta = \theta_1$

Mekanisme runtuh

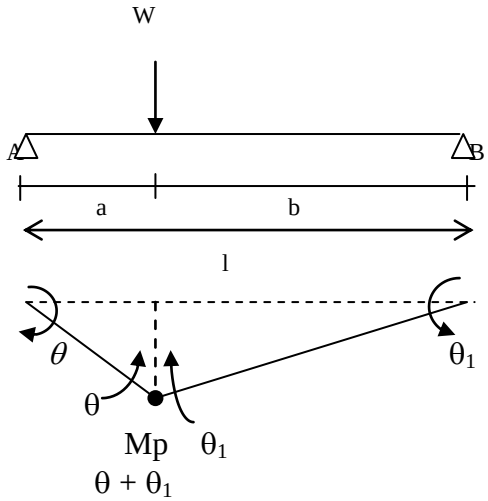
aha luar = $W . \delta = W \frac{1}{2} . \theta . L$

aha dalam = $Mp . 2\theta$

Kesetimbangan : $W . \frac{1}{2} L \theta = Mp . 2 \theta$

jadi : $Wc = \frac{4Mp}{L}$

Contoh : Balok sederhana (beban sembarang)



Mekanisme runtuh

Usaha luar = w. δ

= w. a. θ

Usaha dalam = Mp (θ + θ1)

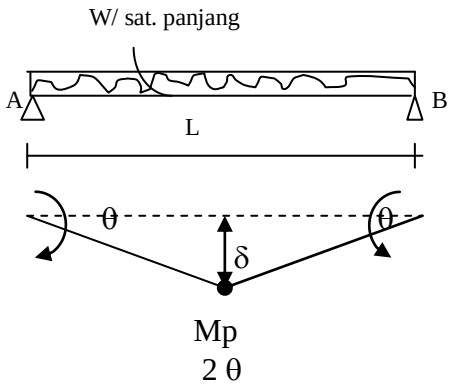
= Mp $(\theta + \frac{a}{b}\theta)$

= Mp $\theta \left(\frac{b+a}{b}\right) = Mp.\theta\frac{L}{b}$

Persamaan kesetimbangan : W_c a. θ = Mp. θ. L/b

$W_c = \frac{M_p.L}{a.b}$

Contoh : Balok sederhana (beban merata)



$\delta = ½ . L. \theta$

Usaha luar $= W. \delta. ½$

$= W. ½ L. \theta. ½$

$= ¼ W. L. \theta$

Usaha dalam $= Mp. 2 \theta$

Kesetimbangan

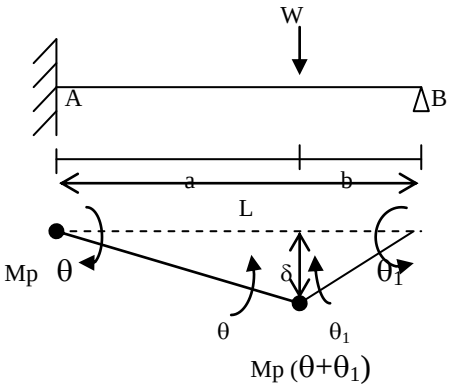
$= ¼ Wc. L. \theta = Mp. 2\theta$

$Wc = \frac{8Mp}{L}$

Atau : usaha luar = $besarnyabeban / unit\ length$ x luas diagram mekanisme runtuh dibawah beban

$= \frac{Wc}{L} x\left(\frac{1}{2}.L.\delta\right) = \frac{Wc}{L} x \frac{1}{2}.L.\frac{1}{2}.L.\theta = \frac{Wc.L.\theta}{4}$

Contoh : Balok jepit – sendi (beban titik)



$\delta = a. \theta = b. \theta_1$

$\theta_1 = a/b . \theta$

Usaha luar $= W . \delta$

$= W . a. \theta$

Mekanisme runtuh

Usaha dalam $= Mp (\theta+\theta_1) + Mp . \theta$

$= Mp \theta + Mp \theta_1 + Mp \theta$

$= 2 Mp. \theta + Mp\ a/b. \theta$

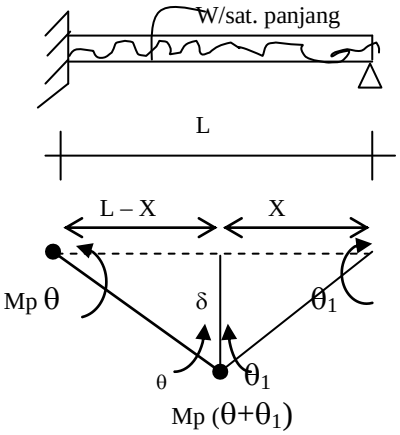
Persamaan kesetimbangan : Usaha luar = usaha dalam

$W . a . \theta = 2 Mp. \theta + Mp. a/b. \theta$

$Wc = \frac{2Mp + Mp.a/b}{a} = \frac{(2 + a/b)}{a} Mp = \frac{2b + a}{b.a} Mp$

$= \frac{b + b + a}{b.a} Mp = \frac{b + L}{b.a} Mp$

Contoh : Balok jepit – sendi (beban merta)



$\delta = (L - x)\,\theta = x\,\theta_1$

$\theta_1 = \dfrac{(L-x}{x}\theta$

Usaha luar = beban x ratarata

= Wc. L x ½ (L-x) θ

= ½ Wc L (L – x) θ

Usaha dalam = Mp. θ + Mp (θ+θ_1)

= Mpθ+ Mp (θ+ $\dfrac{L-x}{x}\theta$)

= Mp (1 + 1+ $\dfrac{L-x}{x}\theta$)

= Mp ($\dfrac{L+x}{x}$)θ

Usaha luar = usaha dalam

½ Wc L (L – x) θ = Mp ($\dfrac{L+x}{x}$)θ

$Mp = \dfrac{Wc.L}{2}.\dfrac{x(L-x)}{L+x}$ $\qquad d\dfrac{u}{v} = \dfrac{v.du - u.dv}{v^2}$

Mp max $\rightarrow$ jika $\dfrac{dMp}{dx} = 0$

$$\frac{dMp}{dx} = 0 = \frac{\frac{WcL}{2}\{(L+x)(1(L-x)+x(-1)) - x(L-x)1\}}{(L+x)^2}$$

(L+x) (L-2x) – x (L – x) = 0 $\rightarrow L^2 - 1\,xL - 2x^2 - xL + x^2 = 0$

$L^2 - 2xL - x^2 = 0$

$x^2 + 2\,xL - L^2 = 0$

$x^2 + 2Xl - L^2 = 0$, maka : x = 0, 4142 L

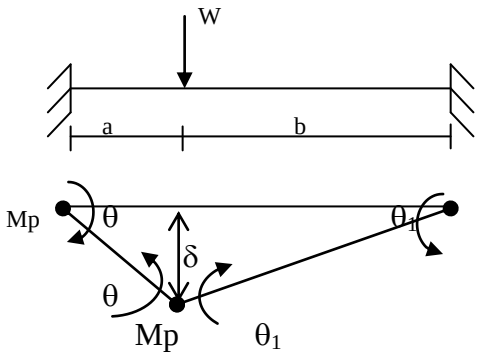
$Mp = \dfrac{Wc.L}{2}.\dfrac{x(L-x)}{L+x}$

$= \dfrac{Wc.L}{2}.\dfrac{0{,}4142L(L-0{,}4142L)}{L+0{,}4142}$

$Wc = 11{,}66\,\dfrac{Mp}{L^2}$

Jadi : $Wc = 11{,}66\,\dfrac{Mp}{L^2}$

Contoh : Balok jepit - jepit (beban titik)



$\delta = a\,\theta = b\,\theta_1$

$\theta_1 = a/b\,\theta$

Usaha luar = $Wc \cdot \delta = Wc \cdot a\,\theta$

Mekanisme runtuh:

Usaha dalam:

$= Mp.\,\theta + Mp.\,\theta_1 + Mp\,(\theta+\theta_1)$

$= Mp.\,\theta + Mp.\,a/b\,\theta + Mp\,(\theta + a/b\,\theta)$

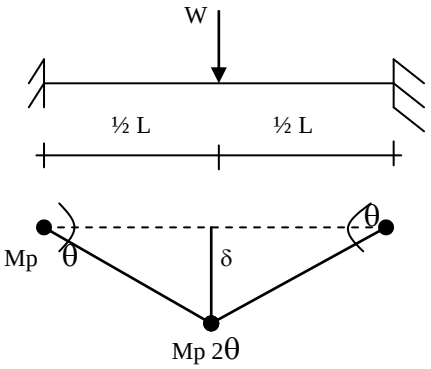
$= 2\,Mp\,(\theta\,(1+a/b)$

$= 2\,Mp\theta\left(\frac{a+b}{b}\right)$

UL = UD $\rightarrow W_C \cdot a\,\theta = 2\,Mp\,\theta\left(\frac{a+b}{b}\right)$

$W_C = 2\,Mp \cdot \frac{L}{a.b}$

Contoh : Balok jepit – jepit (beban titik di tengah)



$\delta = ½\,L.\,\theta$

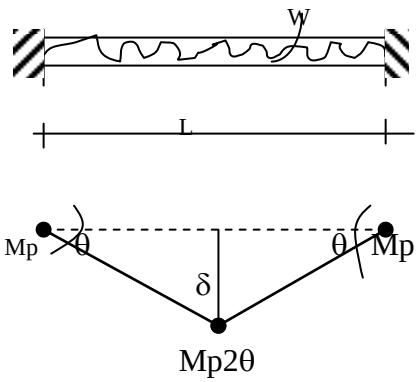
UL $= Wc.\,\delta = Wc.\,½\,L.\,\theta$

UD $= 2\,Mp\,\theta + Mp.\,\theta + Mp\theta$

$= 4\,Mp.\,\theta$

UL = UD $\rightarrow Wc = \frac{4Mp}{\frac{1}{2}L} = \frac{8Mp}{L}$

Contoh: Balok jepit – jepit (beban merata)



UL $= \frac{Wc}{L} x\left(\frac{1}{2}L\frac{1}{2}\theta.L\right) = \frac{Wc.L.\theta}{4}$

Atau:

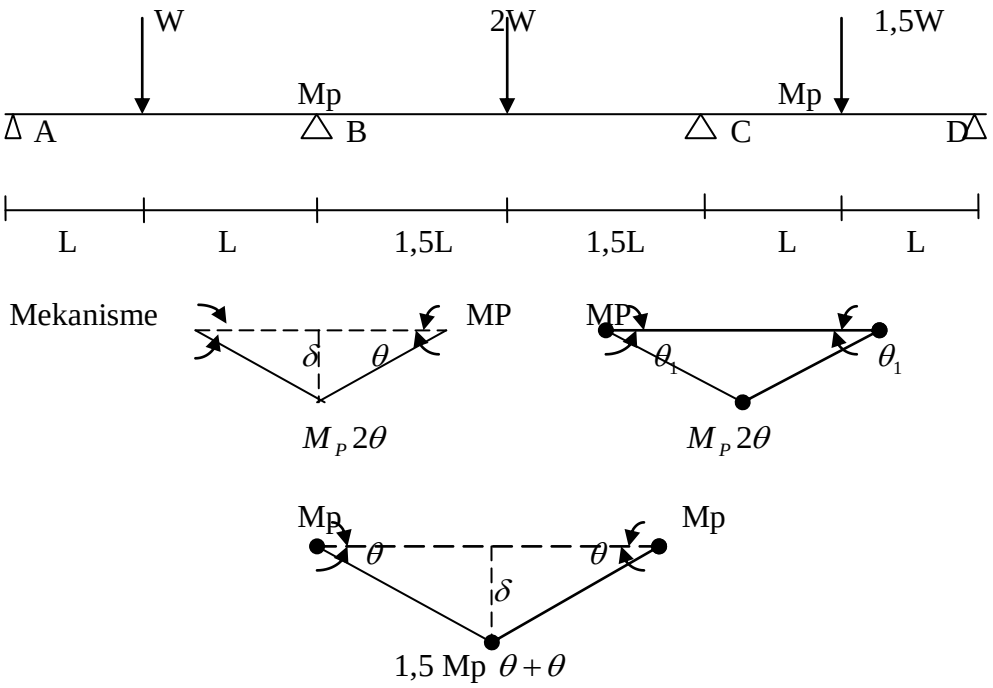
UL = ½ . Wc. δ = ½ Wc. ½ L. θ

$= \frac{Wc.L.\vartheta}{4}$

UD = 4 Mp θ

UL $= UD \rightarrow Wc = \frac{4Mp}{\frac{1}{4}L} = 16\frac{Mp}{L}$

Contoh: Balok menerus (beban titik)



Pada balok A B→ UL = Wc. L. θ

UD = Mp. $2\theta + Mp.\theta$ $\Rightarrow Wc = \frac{3Mp}{L}$

Pada balok CD → UL = 1,5 Wc . L. θ_1

UD = Mp . θ_1 + Mp. $2\theta_1$ $\Rightarrow Wc = \frac{3Mp}{1{,}5L} = \frac{2Mp}{L}$

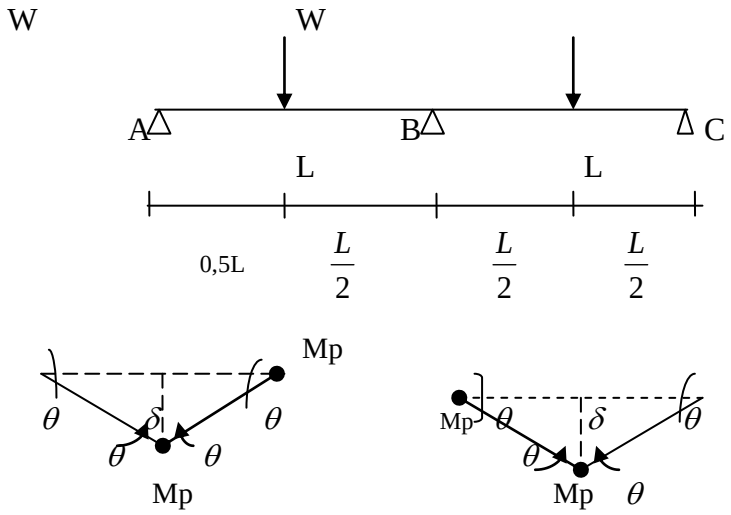
Pada balok BC → UL = 1,5 Wc . 1,5 L. θ

UD = Mp. θ +Mp. θ + 1,5Mp. 2θ $\Rightarrow Wc = \frac{5Mp}{3L} = \frac{5}{3}Mp/L$

$\theta = \frac{\delta}{1{,}5L}$

Yang terkecil Wc = $\frac{5}{3}Mp/L$ (yang menentukan)

Contoh



AB → UL = Wc . $\delta = Wc.\theta.0{,}5L$

UD = Mp. 2θ + Mp. θ = 3 Mp. θ

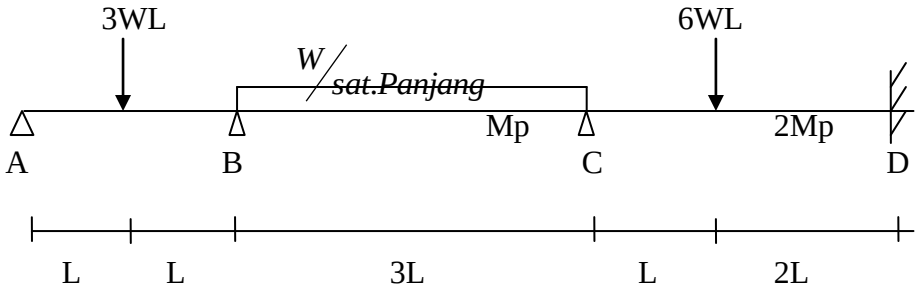
$$Wc = \frac{6Mp}{L}$$

BC → UL = Wc . $\delta = Wc.\theta.0{,}5L$

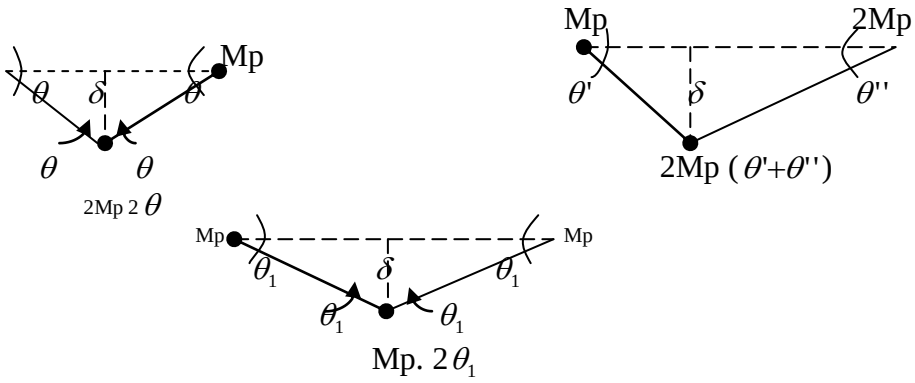
UD = Mp. 2θ + Mp. θ = 3 Mp. θ

$$Wc = \frac{6Mp}{L}$$

Contoh balok menerus (Beban merata)



Mekanisme runtuh



Balok AB → UL = (3 WL). θ . L

UD = 2 Mp. 2θ + Mp. θ = 5 Mp. θ

UL = UD→ Wc= $\frac{5Mp}{3L^2} = 1{,}67Mp/L^2$

Balok BC → UL = $W.\frac{1}{2}(3L).\theta_1.1\frac{1}{2}L = 2\frac{1}{4}.W.L^2.\theta_1$

UD = Mp. θ_1 + Mp. 2 θ_1 + Mp. θ_1 = 4 Mp. θ_1

UL = UD → Wc = $\frac{4Mp}{2\frac{1}{4}.L^2}$ = 1,78 Mp/L^2

Balok CD → UL = (6WL) . δ , δ = θ'. L = θ". 2L

UL = 6W.L. θ'. L θ". = $\frac{L}{2L}\theta' = \frac{1}{2}\theta'$

UD = Mp. θ' + 2Mp (θ'+θ")= 2 Mp. θ"

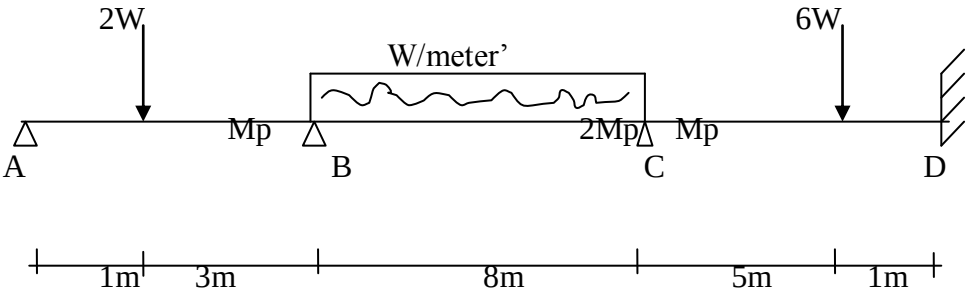
= Mp. θ' + 2 Mp (θ' +1/2 θ')+ 2 Mp ½ θ'

= 5 Mp. θ'

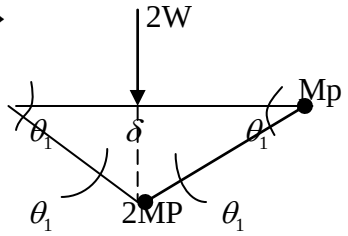
$$UL = UD \rightarrow Wc = \frac{5Mp}{6L^2} = 0{,}83Mp/L^2$$

Jadi beban runtuh Wc = 0,83 Mp/ L^2 (terkecil)

Dimana bentang CD merupakan bentang kritis



AB→



δ = $\theta_1 . 1 = \theta . 3$

$$\theta = \frac{\theta_1}{3}$$

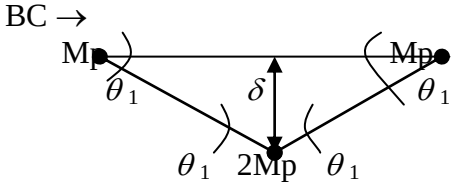
UL = 2W. δ = 2W. θ_1 . 1 = 2W. θ_1

UD= MP. ($\theta_1 + \theta$) + Mp. θ

$$= Mp\left(\frac{4}{3}\theta_1 + \frac{1}{3}Mp\theta_1 = \frac{5}{3}Mp\theta_1\right.$$

$$Wc = \frac{5}{6Mp} = 0{,}83 \text{ Mp}$$

BC →



δ = 4 . θ_1

UL = Wc. ½. δ . 4. θ_1 = 16 Wc. θ_1

Cara lain: beban x ½ δ

= W. 8. ½ . 4. θ_1

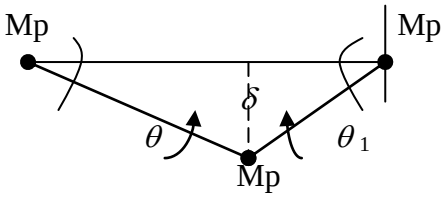
= 16 W. θ_1

UD = Mp. θ_1 + 2 Mp (θ_1 + θ_1 + Mp. θ_1

= 6Mp θ_1

Wc = $\frac{6}{16} Mp = 0{,}37Mp$

CD →



δ = 5. θ = 1. $\theta_1 \rightarrow$ $\theta_1 = 5\ \theta$

UL = 6W. δ = 6 Wc. 5θ

UD = Mp θ + Mp ($\theta + \theta_1$) Mp (θ_1)

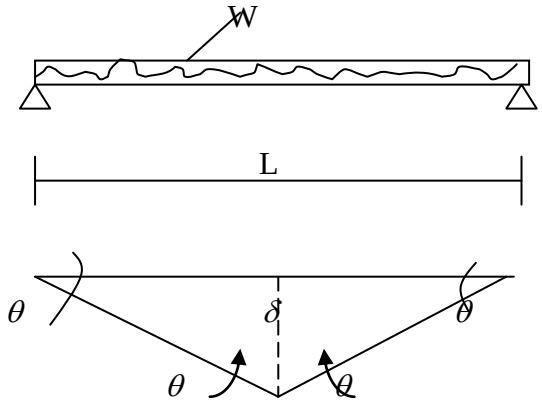
= Mp θ + Mp (6θ) + Mp 5θ

= 12 Mp

Wc = $\frac{12Mp}{30} = \frac{2}{5} Mp = 0{,}4Mp$

Catatan Untuk Beban Merata

1)



δ = ½ L. θ

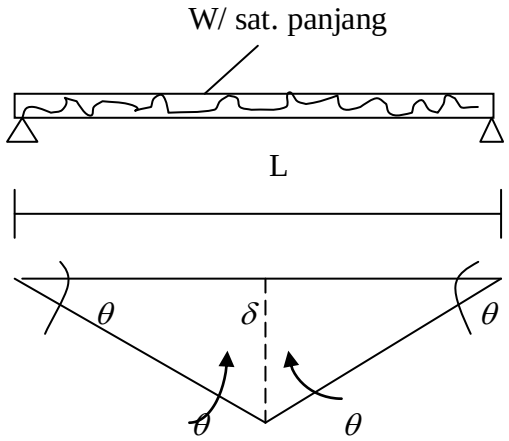
Rata-rata perpindahan dari beban merata w yaitu = ½ δ , jadi

Usaha luar = W. ½ δ = W. ½ . ½ L. θ

= $\frac{WL\theta}{4}$

Usaha luar = beban x rata-rata defleksi

2) $\delta = ½ . L. \theta$

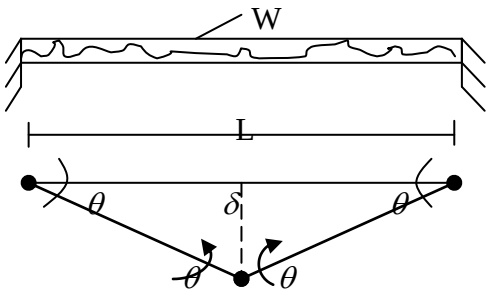


Usaha Luar = besar beban/ sat panjang x diagram mekanisme runtuh di bawah beban

$$UL = \frac{W}{L} x \frac{1}{2} . L.S$$

$$= \frac{W}{L} x \frac{1}{2} . L. \frac{1}{2} . L.\theta = \frac{WL\theta}{4}$$

3)



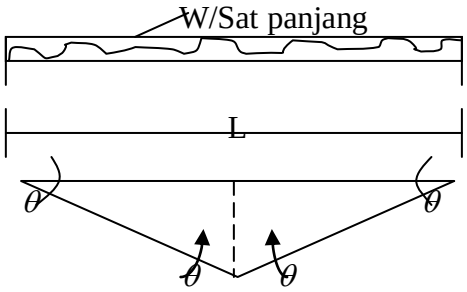
δ $= ½ . L. \theta$

UL $= W. ½ \delta$

$= W. ½ . \delta /2. L. \theta$

$= \frac{WL.\theta}{4}$

4)

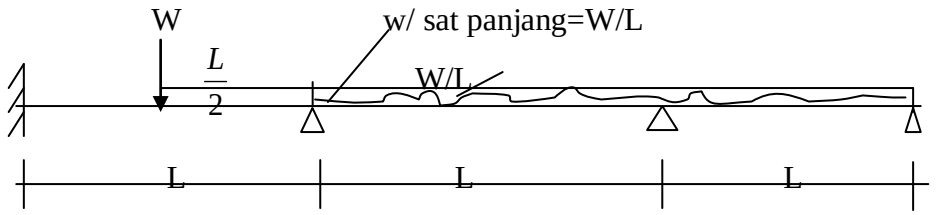


δ $= ½ . L. \theta$

UL $= \frac{W}{L} . \frac{1}{2} L.\delta = \frac{W}{L} . \frac{1}{2} L. \frac{1}{2} L.\theta$

$= \frac{WL.\theta}{4}$

Contoh:

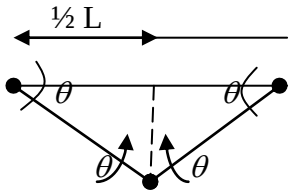


AB → UL = W. $\frac{L}{2}\theta$

UD = Mp. θ + Mp. $(\theta+\theta)$ +Mp. θ

= 4 Mp. θ

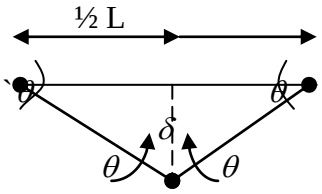
WC = $\frac{4Mp}{L/2}$ = 8Mp/L terkecil



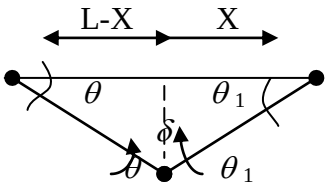
BC → UL = $\frac{W}{L}.\frac{1}{2}L.\left(\frac{1}{2}L.\theta\right)=\frac{WL.\theta}{4}$

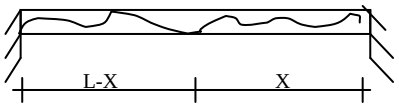
UD = 4 Mp. θ

WC = $16\frac{Mp}{L}$



Batang CD seperti jepit sendi





X= 0,4142L

δ = 0,4142L. θ_1 = 0,5858 L θ atau $\theta_1 = \frac{0,5858}{0,4142}\theta$

UL = beban rata-rata defleksi

= $\frac{W}{L}$. L x ½ x 0,5858 L θ = ½ . 0,5858 WL θ

UL = $\frac{W}{L}$. ½ L. O,5858L. θ = ½ . 0,5858 WL. θ

UD = Mp. θ + Mp $(\theta+\theta_1)$

$$= Mp.\ \theta + Mp\ (1+ \frac{0,5858}{0,4142})\ \theta$$

$$= 3,4143\ Mp.\ \theta$$

Jadi Wc = 11,6569 Mp/ L

Jadi Wc (terkecil) = 8 Mp/ L