GAYA HIDROSTATIK PADA BIDANG LENGKUNG

Gaya yang bekerja pada suatu elemen dengan permukaan lengkung (curvature) perunit lebar dF = w.h.ds

Gambar

Komponen horizontal

- $dF_X = dF.\sin\theta = w.h.ds.\ \sin\theta \rightarrow ds.\ \sin\theta = dh$
  $dF_X = w.h.dh \rightarrow F_X = \int dF_X = ½\ .w.h^2$

Komponen vertikal

- $dF_y = dF.\cos\theta = w.h.ds.\ \cos\theta \rightarrow ds.\ \cos\theta = dh$
  $dF_y = w.h.dx$
  dF_y adalah berat volume fluida diatas elemen ds. Gaya vertical total adalah berat fluida diatas permukaan elemen hingga ke permukaan
  $F = (F_x^2 + F_y^2)1/2$ →resultan gaya
  $$\partial = \tan^{-1}\frac{Fy}{Fx}$$
  Lokasi resultan gaya
  Gambar

  Resultan gaya bekerja pada pusat tekanan
  Gambar

Contoh

Suatu pintu air (gate) dengan lebar w = 5m. persamaan permukaan x =y^2/a (a =4m). kedalaman air disebelah kanan pintu D=4m. hitung komponen gaya horizontal dan vertical dari resultan gaya akibat air serta titik tangkapnya

Gambar

Solusi

Gambar

Persamaan dasar $F_H = \int_0^D p.w.dy$

$F_V = \int_0^{D^{2/3}} p.w.dy$

$F = \int p.dA \rightarrow \frac{dp}{dh} = \rho.g$

Penyelesaian integrasi, diperlukan ekspresi P(y) dan P(x) sepanjang permukaan pintu.

$$\frac{dP}{dh} = \rho.g\ ,\ dP = \rho.g.dh,\ \int_{Pa}^{P} dp = \int_0^h \rho.g.dh$$

P = Pa +ρ.g.h, Pa ≈ 0 → P = ρ.g.h

h = D – y →x =y^2/a Y = √a.$x^{1/2}$

h = D - √a.$x^{1/2}$ → substitusikan ke Fn dan Fu

$F_H = \int_0^D p.w.dy = \int_0^D \rho.g \cdot h \cdot w.dy = \rho.g \cdot w \int_0^D h.dy = \rho.g \cdot w \int_0^D (D-y).dy$

$$= \rho.g \cdot w \left[Dy - \frac{y^2}{2} \right]_0^D = \rho.g \cdot w \left[D^2 - \frac{D^2}{2} \right] = \frac{\rho.g \cdot w \cdot D^2}{2}$$

= 1000kg/m^3 .9,81 m/s^2.5m.4^2/2m^2 = 392400N

F_H =392,4kN

$F_V = \int_0^{D^{2/a}} p.w.dx = \int_0^{D^{2/a}} \rho.w.h \cdot dx = \rho.w.g \int_0^{D^{2/a}} .h \cdot dx$

$$= \rho.w.g \int_0^{D^{2/a}} .h \cdot (D - \sqrt{a} X^{1/2}) dx = \rho.w.g \left[Dx - \frac{2}{3} \sqrt{a} . X^{3/2} \right]$$

$$= \rho.w.g \left[\frac{D^3}{a} - \frac{2}{3} \sqrt{a} . \frac{D^3}{a^{3/2}} \right] = \rho.w.g \left[\frac{D^3}{a} - \frac{2}{3} a^{-1} . D^3 \right]$$

$$= \rho.w.g \left[\frac{3D^3 - 2D^2}{3a} \right] = \frac{\rho.w.g \cdot D^3}{3a}$$

= 1000kg/m^3 .9,81 m/s^2.5m.4^3/3.4m^2 = 261600N

= 261,6kN

Untuk mencari garis kerja F_H, momen F_H terhadap O = jumlah momen dF_H terhadap O

$Y'F_H = \int_{Ax} Y.p.dAx$ dan $Y' = 1/F_H \int_{Ax} Y.p.dAx$

$Y' = 1/F_H \int_0^D Y.p.w.dAy = 1/F_H \int_0^D Y.p.g.h..w.dy = \frac{w.\rho.g}{F_H} . \int_0^D Y.(D-y)..dy$

$$Y' = \frac{w.\rho.g}{F_H} . \left[\frac{D}{2} y^2 - \frac{y^3}{3} \right]_0^D = \frac{w.\rho.g.D^3}{6F_H}.$$

$= \frac{w.\rho.g.D^3}{6} . \cdot \left[\frac{2}{w.\rho.g.D^2} \right]$ =D/3 =4/3 = 1,33 m

Garis kerja F_V, momen F_V terhadap O = ∑ momen dF_V terhadap O

$X'F_V = \int_{Ay} Y.p.dAy$ dan $X' = 1/F_V \int_{Ay} Y.p.dAy$

$X' = 1/F_V \int_0^{D^{2/a}} x.p.w.dAx = 1/F_V \int_0^{D^{2/a}} x.p.g.h..w.dx = \frac{w.\rho.g}{F_V} . \int_0^{D^{2/1}} x.(D - \sqrt{a}.X^{1/2}).dx$

$$= \frac{w.\rho.g}{Fv}.\left[\frac{D}{2}x^2 - \frac{2}{5}\sqrt{a}X^{5/2}\right]_0^{D^{2/a}}$$

$$= \frac{w.\rho.g}{Fv}.\left[\frac{D^5}{2.a^2} - \frac{2}{5}\sqrt{a}\,\frac{D^5}{a^{5/2}}\right] = \frac{w.\rho.g.D^5}{10.Fv.2.a^2}$$

$$= \frac{w.\rho.g.D^5}{10.a^2}\left[\frac{3a}{w.\rho.g.D^3}\right] = \frac{3D^2}{10a} = \frac{3}{4}.\frac{4^2}{4} = 1{,}2m$$