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DEVELOPING MATHEMATICAL UNDERSTANDING INSTRUMENTS FOR SECONDARY SCHOOL STUDENTS

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Abstract

One important step in research is the development of instruments. An instrument should be valid and reliable in order that the conclusions that can be generated from research can be used successfully. This paper presents the instrument development of mathematical understanding for secondary school students in the form of essay test. The instrument development steps include: (1) studying the literature about mathematical understanding; (2) making a syllabus based on curriculum, learning material, the indicators of mathematical understanding, the characteristics of students and learning approach; (3) making test items; (4) content and face validation; (5) revision based on the results of the content and face validation; (6) trial; (7) analysis of trial results. From the analysis will produce conclusions, namely a set of mathematical understanding instruments that are valid and reliable, for junior high school students consisting of 5 items.

Keywords: mathematical understanding instrument, mathematical education

Introduction

One of the goals of KTSP for mathematics subjects at secondary school level is that learners have the ability to understand mathematical concepts, explain the connection between concept and apply the concepts or algorithms in solving problems. It shows that the ability of mathematical understanding is one of the abilities that need to be developed in mathematics at the secondary school level.

In NCTM 2000 mentioned also that mathematical understanding is a very important aspect in the principles of mathematics learning. Students in learning mathematics must be accompanied by an understanding, it was a vision of learning mathematics. It is expressed also in the NCTM 2000 that learning without understanding is the things that happen and be a problem since the 1930s, so learning with understanding is increasingly emphasized in the curriculum.

Fact in knowable field that mathematical understanding ability of Indonesia student especially secondary school student still be low, this thing is visible from result of UN mathematics assessing it is lower relative compared to UN result of the other study area. At international level, achievement of mathematics of the Indonesia students also still low. Study result indicates that achievement of Indonesia school

students mathematics left behind from achievement of school student mathematics in some neighbor states. For example, from result of study TIMSS (Trends in International Mathematics and Science Study) the year 1999 achievement of student mathematics we stay at sequence to 34 of 38 states participating, in the year 2003 residing in at sequence to 36 of 45 states participating, while in the year 2007 residing in at sequence to 36 of 49 states participating. This far achievement below (under achievement of students from neighbour state like Singapore, Malaysia and Thailand, where third of the state at TIMSS the year 2007 each resided in third sequence, 20^{th} sequence and 29^{th} sequence).

Based on the background that have been explained above, hence to develop the mathematical understanding ability of secondary school student, research on mathematical understanding should be done. To support these research, in this paper will explain about the development of mathematical understanding instruments and analysis trial results that have been made to the instrument.

Mathematical Understanding

The expert of mathematics education agree that one of purpose of mathematics learning is comprehend mathematics. The thing causes that in every study of element mathematics there must be understanding of mathematics. In NCTM 2000 it is mentioned that mathematical understanding is a real important aspect in mathematics study principle. Student in learning mathematics must be accompanied with understanding, this thing is vision from mathematics learning. The thing becomes emphasis NCTM, because practically, learnt without understanding was the happened and becomes problem since year 1930 (NCTM, 2000). Mayer; Olsson & Rees; Perkins&Simmons (Dahlan, 2004:46) mentions that understanding is fundamental aspect in study, causing study model must figure in the fundamental from understanding. Remembers the importance of mathematical understanding ability and reality that mathematical understanding ability of student still be low, hence the ability still need to be improved in mathematics study.

Skemp (1976) differentiates two understanding type, there are instrumental understanding and relational understanding. Instrumental understanding a number of concepts interpreted as understanding to concept that is each other separate and only memorizes formula and apply it in calculation without reasons. On the contrary at relational understanding included a complex scheme or knowledge structure and is each other correlates which can be applied at solving of broader problem and complex.

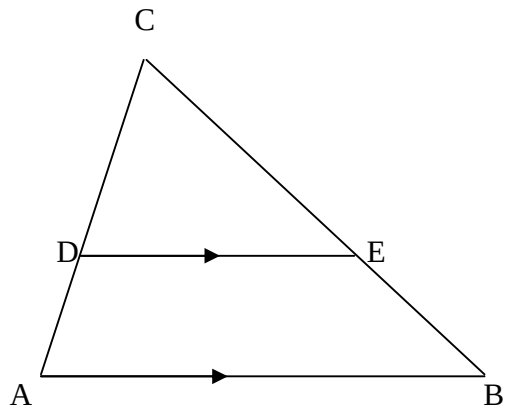
Identical with Skemp (1976) opinion is expressing that there are two understanding types that is : instrumental and relational, Hiebert arises the opinion about procedural knowledge identical with instrumental understanding, and conceptual knowledge identical with relational understanding. However, between Skemp and Hiebert there is difference about relation between two the abilities. Even and Tirosh (2002) express that Skemp gives boundary that is clear between two the abilities so that there is dichotomy between instrumental understandings and relational understanding. While Hiebert doesn't give assertive boundary between procedural knowledge and conceptual knowledge, so that between two abilities is in character continue.

Experts differ in opinion about instrumental understanding and relational understanding. Reys (1998:21) in the middle of existence of different idea of the experts about which is more between procedural understandings and conceptual understanding lays open that both the understandings is important in mathematics expertise. Procedural understanding is based on sequence of actions, often involving rules and algorithms; conceptual understanding, on the other hand, is based on connected networks that link reationships and discrete pieces of information (Hiebert and Lefevre, in Reys, 1998:21), where this thing also hardly required in mathematics learning.

Other opinion about understanding told by Bloom (Wikipedia, 2009), is expressing that there are 3 kinds of understanding that is: translation, interpretation, and extrapolation. Implementation of that understanding in mathematics can be explained as follows: translation, for example can change an equation to become a graph, solvent of problem deluge is in the form of words becomes form of symbol or on the contrary. Interpretation, for example can determine correct concepts to be used in finalizing problem, can interpret an equality. While extrapolation, for example can apply concepts in mathematical calculation, can estimate tendency a diagram.

Anderson & Krathwohl (2001:70) in revised Bloom's taxonomy state that the cognitive process of understanding is 7, namely:

1. Interpreting : Changing from one representation to another representation. Example: In the following picture is known that AB parallel to DE.



Students can interpret that:
$\angle CDE = \angle CAB$ and $\angle CED = \angle CBA$

2. Exemplifying/Ilustrating: Finding a specific example or illustration of a concept Example: Students can give examples of two triangles are congruent.
3. Classifying: Specifies that an instance or a case included in the category of a concept or not.
   Example: Served a variety of drawing a triangle. From the presented triangle images, students can find a congruent triangle and which are not congruent.
4. Summarizing, generalizing: Make a statement that represents some of the information presented.
   Example: Given an image of two triangles are similar as follows:

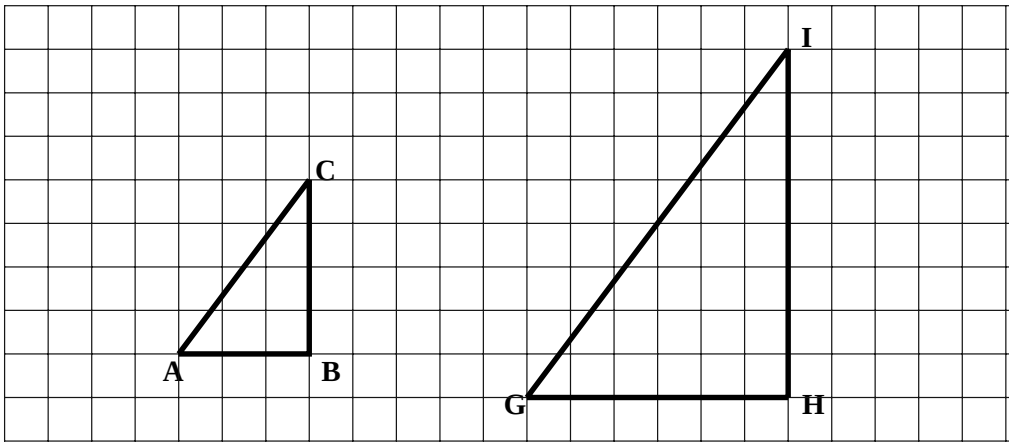


Figure 1
Two similar triangles

with information from these images the students are directed to observe the length ratio of corresponding sides between the two triangles. From the data obtained, students may conclude that two triangles have the same side length ratio.

5. Inferring : Finding patterns from a collection of examples or cases. Example: Presented various images of congruent triangles in different sizes. From the presented triangles images, students can expect that congruent triangles have congruent angles.
6. Comparing : Detects similarities and differences between two objects or more.

Example: Served a variety of triangles, students can detect similarities and differences in the triangles are presented.

7. Explaining: Constructing and using a cause and effect system of a concept. Example: Students can explain the concept of similar triangles.

Based on the explanation above, the ability of mathematical understanding will be seen through the students' ability in solving mathematical problems. In every mathematical problem solving, aspects of mathematical understanding is measured through indicators of: classifying the objects of mathematics; interpret ideas or concepts; find examples of a concept, give examples and not an example of a concept, and expressed the concept of mathematics with their own language.

Understanding of mathematical tests that have been developed can be seen in appendix, while the grid presented in Table 1

Table 1.

Mathematical Understanding Problem Grid

Level: Junior High School
Subject: Mathematics
Grade/Semester: IX / 1

Material	Understanding Aspects Measured	Indicator	Number
Similar Triangles	Provide examples and not an example of a concept	From the pictures provided, students can give examples and non examples of similar triangles.	1.a
	Classify the objects of mathematic	Students can classify the corresponding sides.	1.b
	Stating the concept of mathematics with their own language.	Students can express mathematical concepts that underlie their answer	1.c
Congruent Triangles	Finding examples of the concept. Stating the concept of mathematics with their own language	From the pictures provided, students can find specific examples of the concept of congruent triangles. Students may declare mathematical concept that underlies the answer given.	2.a
	Classify the objects of mathematics	Students can classify the triangle sides of the same length.	2.b
Volume of Spheres and Cylinders	Stating the concept of mathematics with their own language.	Students can express mathematical concepts underlying the change in volume in the cylinder caused by the entry of the sphere in the cylinder.	3.
Volume of Cylinders	Interpret ideas or concepts	Students may interpret the ideas associated with volume changes in the cylinder and the students can express the concept of volume of cylinder.	4.
Volume of Cones	Interpret ideas or concepts	Students can interpret ideas related to the volume of cone	5.a
	Stating the concept of mathematics with their own language	Students can express the concept of volume of a cone.	5.b

Result Analysis and Discussion

The Validity of Mathematical Understanding Instruments

Mathematical understanding instrument validity test through consideration of the experts about the content and the face of the test in mathematical understanding. The results of consideration are presented in Table 2 and Table 3.

Table 2
Result of Face Validity Considerations of Mathematical Understanding Test

Problem Number	Validator				
	1	2	3	4	5
1.	1	1	1	1	1
2.	1	1	1	1	0
3.	1	1	1	1	0
4.	1	1	1	1	1
5.	1	1	1	1	1

Note: 1 = Valid, 0 = Not Valid

Table 3
Result of Content Validity Considerations of Mathematical Understanding Test

Problem Number	Validator				
	1	2	3	4	5
1.	1	1	1	1	1
2.	1	1	1	1	0
3.	1	1	0	1	0
4.	1	1	1	1	1
5.	1	1	1	1	1

Note: 1 = Valid, 0 = Not Valid

Experts considerations presented in the table above then analyzed using Q-Cochran statistical test. Statistical test results of consideration of face validity is presented in Table 4, while the result of consideration of content validity is presented in Table 5.

Table 4
Q-Cochran test of the face validity of Mathematical Understanding Tests

N	5
Cochran's Q	8.000(a)
Df	4
Asymp. Sig.	.092

a 1 is treated as a success.

Table 5
Q-Cochran test of the face validity of Mathematical Understanding Tests

N	5
Cochran's Q	6.400(a)
Df	4
Asymp. Sig.	.171

a 1 is treated as a success.

Based on Table 4 and Table 5 above shows that the price of Q-Cochran statistics for face validity and content validity were 8.000 and 6.400 and asymtotic significance were 0.092 and 0.171. Because the value of asymptotic significance greater than 0.05 then it can be concluded that the validator are giving equal consideration to the face validity and equal consideration to the content validity of the mathematical understanding with the significance level of 5%.

The Reliability of Mathematical Understanding Instruments

To know the reliability of mathematical understanding tests used a Cronbach Alpha statistical tests. Trial test results of mathematical understanding test presented in Table 6.

Table 6

Trial Results of Mathematical Understanding Test

Students	Problem Number				
	1	**2**	**3**	**4**	**5**
UPM1	2	1	1	2	2
UPM2	2	1	2	3	1
UPM3	3	3	2	3	3
UPM4	3	1	2	2	1
UPM5	4	2	3	1	2
UPM6	1	1	2	2	1
UPM7	4	3	3	3	3
UPM8	3	2	3	3	4
UPM9	3	2	2	3	2
UPM10	4	3	2	4	4
UPM11	3	1	3	2	1
UPM12	3	3	3	2	2
UPM13	2	1	2	2	2
UPM14	5	3	3	4	3
UPM15	4	2	2	3	3
UPM16	2	1	1	2	1
UPM17	5	2	3	3	3
UPM18	2	1	1	1	0
UPM19	3	1	2	1	0
UPM20	4	2	2	1	2
UPM21	2	2	0	0	0
UPM22	3	3	0	0	0
UPM23	3	3	2	0	0
UPM24	2	4	4	0	3
UPM25	3	4	3	0	3
UPM26	3	3	2	2	1
UPM27	3	3	2	2	1

Students	Problem Number				
	1	2	3	4	5
UPM28	3	3	3	2	2
UPM29	4	3	3	4	4
UPM30	3	3	3	2	3
UPM31	4	3	2	2	2
UPM32	4	1	1	3	2

Based on calculations using the SPSS 15, from Table 5 that the test result has a reliability coefficient of Mathematical Understanding of 0.751 as shown in Table 7.

Table 7
Mathematical Understanding Test Reliability

Cronbach's Alpha	N of Items
.751	5

According Guilfrod (in Ruseffendi, 2005), the instrument is considered reliable enough when $r > 0.70$. Since $r = 0.751$, then the mathematical undertanding test are reliable enough.

Differentiator Power of Mathematical Understanding Test

From the data of mathematical understanding test trials, the results of **d**ifferentiator power calculations are shown in Table 8 as follows:

Table 8
Results and Interpretation of Differentiator Power
of Mathematical Understanding Test

Problem Number	Differentiator Power	Interpretation
1.	0.333	Sufficient
2.	0.222	Sufficient
3.	0.361	Sufficient
4.	0.528	Good
5.	0.667	Good

Difficulty Level of Mathematical Understanding Test

The Results of calculation about the difficulty level from the data of mathematical understanding test trials are shown in Table 9 as follows:

Table 9
Results and Interpretation of Difficulty Level
of Mathematical Understanding Test

Problem Number	Difficulty level	Interpretation
1.	0.619	Sufficient
2.	0.444	Sufficient
3.	0.539	Sufficient
4.	0.500	Sufficient
5.	0.477	Sufficient

Student Difficulties in Mathematical Understanding Tests

From the mathematical understanding test trials, student difficulties in mathematical understanding tests are shown in Table 10 as follows:

Table 10
Student Difficulties in Mathematical Understanding Tests

Problem Number	Material	Student Difficulties
1	Similar Triangles	In this number, students did not experience significant difficulties. But in general students are less able to convey arguments systematically.
2	Congruent Triangles	In general, many students are stuck on an image that seems congruent (but not congruent). The students are also less able to convey the arguments above are correct answers.
3	Volume of Spheres and Cylinders	At this number, students are much less precise in the calculation. Students in general are also less able to convey arguments systematically.
4	Volume of Cylinders	In this number are many students who have difficulty in understanding the problem and also less able to convey arguments systematically.
5	Volume of Cones	In this number the students much less precise in presenting in the form of pictures and many are wrong in interpreting the problems.

From Table 10 shows that in general, students' difficulties of mathematical understanding test are similar. But the smart students are better able to interpret the problem correctly as well as presenting a variety of representations, and the other students are less able to develop interpretation and less able to provide arguments that are more complex. Students are generally glued to the examples that already exist in the book or who have been given by the teacher.

Conclusion

From the analysis and discussion, it can be concluded that the mathematical understanding tests that have been prepared are valid and reliable, for junior high school students consisting of 5 items. The tests also have a quite good differentiator power and the level of difficulty. Thus the five-point test in mathematical understanding can be relied and used as an instrument of the research.

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